1 Introduction

1.1 Nanophotonics for Functional Devices

With the necessity for higher-speed and large amount of information processing in the near future, miniaturization of optical devices have been progressed[1], which have almost reached a critical limit determined by the diffraction limit of conventional propagating light[2,3]. We have expected since the 1990s that an optical near field is one of important technologies to overcome the limit, and many researches have been performed in various fields, such as fundamental physics in a nanometric space, optical near-field microscopy and spectroscopy, optical measurement, bio-imaging, nanofabrication, and nanophotonic device architecture[4]. As you well know, an optical near field is quite characteristic electromagnetic field which is localized around a nanometric object, and decay length of the field depends on the size of the object, which is smaller than the wavelength of incident light. This size dependence is caused that the optical near field cannot be considered without matter excitation; in a nanometric space, an incident electromagnetic field is modified by matter excitation in a nanometric object, and the modified field also affects itself and another neighboring one before releasing the energy as far-field photons. Such nanometric light-matter interaction must describe as a self-consistent field. We are aiming to realize nanometric functional devices free from the diffraction limit of light, in which such optical near field play roles of information carrier and control signal. We refer to such devices as nanophotonic devices. The readers seem that a localization feature of nanophotonic devices is resemble to electronic devices in which electric charge always stays in the device. However, in a nanophotonic device, the localized
field enables to leave an object and release photons in the far field via optical
near-field interaction among several nanometric objects[5]. We have consid-
ered that an important concept of nanophotonic device and nanophotonic
device operations is to deal with light-matter interaction in a nanometric
system as well as dissipation of matter excitation energy toward the outer
field. Since a signal is finally detected as the far-field light, the dissipation
process needs to be controlled by the nanometric light-matter interaction.
Hence, a nanophotonic device could operate inherent way in nanophotonics,
which is different from conventional optical and electronic devices.

The advantages of nanophotonic devices are not only the miniaturization
but also to possess the possibilities for novel principles of functional device
operations inherent in nanophotonics. As mentioned above, in a nanopho-
tonic device, some characteristic physics are used, such as inherent matter
excited states in optical near-field interaction, coupling between near and
far-field light, and coupling between matter excitation and phonons, a part
of which has not been considered in conventional optics. Therefore, design of
a nanophotonic device might be different from that in the conventional ones
since basic principles differ, and moreover, they can accomplish the func-
tions which have not realized so far. Following this, we need to consider how
a nanophotonic devices should be designed, and, in addition, we need how
nanophotonic devices coexist with the other devices.

In the present chapter, we concentrate our discussions how to use the
nanophotonic inherent features for functional device operations, and consider
what and how we can realize. In the next section, we explain some charac-
teristic features in nanophotonics, and catch a basic outline of nanophotonic
devices.

1.2 Nanophotonic Inherent Features

In general a functional device, the followings are indispensable; preparation
of appropriate input states, propagation of a signal, and control of the signal.
Nanophotonics possesses characteristic features for all of them, which cannot
observe for the far-field light. Here we explain these features inherent in
nanophotonics; locally excited state which cannot create by using far-field
light, unidirectional energy transfer, and dependence of excitation number
in which coupling between discrete energy levels and the optical near field
plays an important role. These regard as key features for nanophotonic device
operations.

Locally Excited States
First, we explain difference of matter excitation of nanometric objects be-
tween by using the far- and near-field light. Figure 1(a) illustrates in the
case that the nanometric objects are irradiated by the far-field light. Since
the objects that are located in the area much smaller than the wavelength
of light are simultaneously excited by a uniform field, we cannot recognize the matter excited state independently in each object. The detected far-field light also has not any information about the excited state in each object. While in the case of optical near-field excitation, which can be accomplished by setting an optical near-field probe, such as a nanometric metallic aperture, an optical fiber probe, and a single molecule, an individual object can be selectively irradiated, and a locally excited state can be created because of existence of the light localized around the probe. Moreover, the excited object makes secondary electromagnetic field, which affects the neighboring one again mediating via the optical near field, and thus, the excited states in this system are determined selfconsistently by the optical near-field interaction. Figure 1(b) schematically shows such a situation of the optical near-field excitation. This asymmetric excitation also influences the far-field light which we can detected as an information signal.

In the following, we describe the excited states discussed above as algebraic expressions. Here we restrict our discussion within coupling of two 2-level systems of excitons, for simplicity (See Fig. 2). For the far-field excitation, the nanometric objects are uniformly excited, and thus, a one-exciton state, in which an exciton exists in the system, can be written as

$$|1\rangle_s = (|e\rangle_A|g\rangle_B + |g\rangle_A|e\rangle_B)/\sqrt{2},$$  \hspace{1cm} (1)

where, $|e\rangle_i$ and $|g\rangle_i$ represent exciton state and crystal ground state, respectively. The subscripts $i = A$ and $B$ are the label of two nanometric objects, and the meaning of subscript $s$ will be mentioned below. Eq. (1) means that the exciton in an isolated system cannot be distinguished because an exciton exists in both object A and object B with equivalent probabilities.

On the other hand, as mentioned above, an optical near field enables to create an exciton in an individual object. The exciton prepared in this system goes and returns between two 2-level systems for a period depending on the strength of optical near-field coupling, which is called near-field optical nutation[6,7]. However, if the pumping time is much shorter than the period of the near-field optical nutation, we can create locally excited states in this system. Such locally excited states with an exciton in the system, can be written by linear combination of coupled states, which extend between two objects, as follows:

$$|e\rangle_A|g\rangle_B = (|1\rangle_s + |1\rangle_a)/\sqrt{2},$$  \hspace{1cm} (2a)

$$|g\rangle_A|e\rangle_B = (|1\rangle_s - |1\rangle_a)/\sqrt{2}.$$  \hspace{1cm} (2b)

The right-hand sides in Eqs. (2a) and (2b) are described by the coupled states via an optical near field, where the subscripts $s$ and $a$ mean symmetric and anti-symmetric states, respectively. The readers easily understand that there are two coupled states for the optical near-field excitation, while the far-field light excites the only symmetric state. Note that we did not show the anti-symmetric state in Eq. (1), since the state is optically inactive for the far-field
light, which can be verified from the following relation, \( a \langle 1 | H_{\text{int}} | g \rangle = 0 \), where \( |g\rangle = |g\rangle_A |g\rangle_B \) and the interaction Hamiltonian refers to Eq. (4). The locally excited states are quite important for functional operations in our proposed nanophotonic devices, which are discussed in both Secs. 3 and 4.

**Unidirectional Energy Transfer**

For functional device operations which manipulate information carrier, an excitation or carrier must transfer unidirectionally from input to output terminals. In conventional optical devices, the unidirectional energy transfer can be accomplished by using an optical isolator which blocks reflected right generally by using polarization of light. Unless we use the polarization, the size of optical devices is restricted by the wavelength of light. In electronic...
devices, unidirectional signal transfer is easily satisfied since electrons flow along an electrical potential. However, when the size of electronic devices becomes smaller and quantum mechanical effects appear, the electrical signal receives a noise because of universal quantum fluctuations. In a nanophotonic device, the signal isolation using wave character of light is impossible because of the diffraction limit of light. A signal carrier is electrically neutral quasi-particles of electrons and holes, and thus, a static electrical potential cannot drive them. However, we have known an effective way to realize unidirectional exciton energy transfer by using relaxation process among quantum discrete energy levels[8]. Figure 3 is schematic image of energy transfer via an optical near-field in the system which consists of two nanometric objects with 2- and 3-energy levels. As mentioned in the previous subsection, the optical near-field coupling makes coherently coupled excited state between $E_1$-level in the 2-level system and $E_2$-level in the 3-level system, which strongly couples when the both energies are equal. If we can drop the excitation into the lower $E_1$-level in the 3-level system before radiative lifetime of $E_1$-level in the 2-level system ($\sim 1$ ns), the excitation is confined in the energy level because of off-resonance, and irreversibility is guaranteed in the nanometric system except for radiation from the energy level. Section 2 devotes detailed discussion about the optical near-field coupling and the energy transfer dynamics. Here we note that $E_2$-level in the 3-level system is generally dipole inactive for the far-field light, and thus, the unidirectional energy transfer can achieved by mediating the optical near field.

Since excitations in dipole active levels, $E_1$-level in the 2-level system and $E_2$-level in the 3-level system, can be excited by external far or near-field light, the energy transfer is controllable in this system. This constructs a simple switching operation by using a state-filling nature of excitons. In Sec. 3, we propose a nanophotonic switch using such energy transfer and state-filling, and evaluate dynamics of excitation both analytically and numerically.

**Dependence of Excitation Number**

Although symmetric and anti-symmetric states in Eqs. (1) and (2) describe the one-exciton states, we can obtain a quite interesting feature by considering a two-exciton state in the system shown in Fig. 2. The two-exciton
Fig. 3. Energy transfer between 2-level and 3-level systems. $E_2$-level in the 3-level system is dipole inactive, and thus, the unidirectional energy transfer is achieved only by mediating an optical near field.

state, in which two excitons completely occupy both two 2-level systems, is algebraically written as

$$|2\rangle_p = |e\rangle_A |e\rangle_B,$$

where the number 2 in the left-hand side means the two-exciton state. It is valuable to investigate energies for all base states, $|1\rangle_s$, $|1\rangle_a$, and $|2\rangle_p$. The Hamiltonian for the two 2-level systems coupled via an optical near-field interaction is given by

$$\hat{H} = \hat{H}_0 + \hat{H}_{\text{int}},$$

$$\hat{H}_0 = \hbar \Omega \hat{A}^\dagger \hat{A} + \hbar \Omega \hat{B}^\dagger \hat{B},$$

$$\hat{H}_{\text{int}} = \hbar U (\hat{A}^\dagger \hat{B} + \hat{A} \hat{B}^\dagger),$$

where $\hat{H}_0$ and $\hat{H}_{\text{int}}$ represent the unperturbed Hamiltonian and the interaction Hamiltonian, respectively. $(\hat{A}^\dagger, \hat{A})$ and $(\hat{B}^\dagger, \hat{B})$ are the fermionic creation and annihilation operators in the 2-level systems A and B, respectively. Since we assume the fermionic excitons, and the optical near-field coupling $U$ generates a completely coherent process, which is explained in detail in Sec. 2.

The energies for the states are given as follows:

$$\langle 1 | \hat{H} | 1 \rangle_s = \hbar (\Omega + U),$$

$$\langle 1 | \hat{H} | 1 \rangle_a = \hbar (\Omega - U),$$

$$\langle 2 | \hat{H} | 2 \rangle_p = 2 \hbar \Omega.$$

From Eqs. (5a) and (5b), we find that the energies of the coupled states, $|1\rangle_s$ and $|1\rangle_a$, depend on the strength of the optical near-field coupling, $U$, and the energy differences from the energy in the isolated 2-level system have opposite contributions in the both states. While in the two-exciton states in Eq. (5c), the energy is apparently degenerated because of the both systems.
Fig. 4. Conceptual structure of nanophotonic devices, which consists of a quantum mechanical part and a classical dissipative part. Quantum mechanical part builds up characteristic excited states, and classical dissipative part identifies certain states and connects to outer detection systems.

being filled completely. These characters are useful for selective energy transfer in nanophotonic devices; sequential logic operations can be realized by using the dependence of the excitation number in this system.

We schematically illustrate a fundamental concept for such a nanophotonic device using the above selectivity in Fig. 4. In the device, the quantum mechanical and classical parts are coexisted; in the quantum mechanical part, some characteristic excited states are created, and then, these states have to be extracted selectively from the quantum mechanical part to classical dissipative one in order to connect a signal to an outer detection system. This is a key process to drive the nanophotonic device. We discuss some functional operations in nanophotonic devices in Sec. 4, which are based on such conceptual structures.

2 Optical Near-Field Coupling

In this section, we give a full account of energy transfer between locally excited states via an optical near field. From our theoretical treatment of optical near-field coupling, the readers will understand why dipole-inactive energy transfer for the far-field light changes allowed transition in the case of the optical near field. We also give concrete numerical results of coupling strength in a CuCl quantum-dot system, which determines operation speed of nanophotonic devices discussed in the following Secs. 3 and 4.

2.1 Theoretical Descriptions of an Optical Near Field

There are two way to describe light-matter interaction theoretically; one is to use the minimal coupling Hamiltonian $p \cdot A[9]$, $p$ being the electronic
momentum and $A$ the vector potential, and the other is to use the multipolar QED Hamiltonian\[10,11\] in the dipole approximation, $\mu \cdot D$, where $\mu$ and $D$ represent the electric dipole moment and electric displacement field, respectively. The two descriptions of light-matter interaction are connected by Power-Zienau-Woolley Transformation\[12\], which is a unitary transformation of the Coulomb gauge Hamiltonian. Here, we use the multipolar QED Hamiltonian because there are several advantages in the multipolar QED; first of all, it does not contain any explicit intermolecular or inter-quantum-dot Coulomb interactions in the interaction Hamiltonian and entire contribution to the fully retarded result is from exchange of transverse photons, while in the minimal coupling, the intermolecular interactions arise both from exchange of transverse photons, which include static components, and from instantaneous intermolecular electrostatic interactions\[13\]. Second, it clarifies physical interpretation of the dipole-inactive transition via the optical near field as we will discuss below.

Since our purpose of discussions are to propose and investigate nanophotonic solid devices, in the following, we assume nanometric objects as quantum dots with discrete energy levels. In order to explain an extremely important feature in nanophotonics, we treat internal electronic structures in a quantum dot regarding as collection of local dipoles, which is convenient to express the interactions between nanometric objects and an optical near field spatially distributed in a nanometric space. We can also depict a dipole in one-body problem by using an effective mass approximation. Such theoretical approach has been already published\[14\] where the enhancement of electric quadrupole coupling was pointed out by assuming steep variation of electric field due to the optical near field. This phenomenon is equivalent to our result of the dipole-inactive transition, but our theoretical formulation, in which field variation is caused by the coupling between the local dipoles in the neighbouring quantum-dot pair, is easy to obtain a physical interpretation.

In the following subsections, we present the interaction Hamiltonian in second-quantized form in terms of electron basis functions satisfying the quantum-dot boundary conditions, as well as transition dipole moments of excitons, and derive an optical near-field coupling on the basis of the projection operator method which is explained in Sec. 2.3.

2.2 Excitation and Transition in a Quantum Dot

Interaction Hamiltonian

According to the dipole coupling in the multipolar QED Hamiltonian, the interaction between photons and nanometric materials can be written as\[11\]

$$\hat{H}_\text{int} = -\int \psi^{\dagger}(r) \mu(r) \psi(r) \cdot \hat{D}(r) dr,$$

where $\psi^{\dagger}(r)$ and $\psi(r)$ denote field operators for electron creation and annihilation, respectively, and the dipole moment and the second-quantized electric
displacement vector at position $\mathbf{r}$ are expressed as $\boldsymbol{\mu}(\mathbf{r})$ and $\hat{D}(\mathbf{r})$, respectively. In a quantum dot, the electron field operators should be expanded in terms of basis functions $\phi_{\nu n}(\mathbf{r})$ that satisfy the electron boundary conditions in a quantum dot, that is analogy to those in bulk materials where the Bloch functions satisfying periodic boundary condition are used. The field operators are given by

$$
\psi(\mathbf{r}) = \sum_{\nu=c,v} \sum_{n} \hat{c}_{\nu n} \phi_{\nu n}(\mathbf{r}),
$$

(7a)

$$
\psi^\dagger(\mathbf{r}) = \sum_{\nu=c,v} \sum_{n} \hat{c}_{\nu n}^\dagger \phi_{\nu n}^*(\mathbf{r}),
$$

(7b)

where $\hat{c}_{\nu n}$ and $\hat{c}_{\nu n}^\dagger$ represent the creation and annihilation operators for the electrons specified by $(\nu, n)$, respectively, and the indices $\nu = c, v$ denote the conduction and valence bands. The discrete energy levels in the quantum dot are labelled $n$. The basis functions satisfy the following completeness condition, as well as orthonormalization:

$$
\sum_{\nu=c,v} \sum_{n} \phi_{\nu n}^*(\mathbf{r}) \phi_{\nu m}(\mathbf{r}') = \delta(\mathbf{r} - \mathbf{r}').
$$

(8)

Simultaneously, we express the electric displacement vector $\hat{D}(\mathbf{r})$ using exciton-polariton creation and annihilation operators $(\hat{\xi}_k^\dagger, \hat{\xi}_k)$, where branch suffix of the exciton-polariton is suppressed by taking only an upper branch. We consider exciton-polaritons because a nanometric system in a near-field optical environment is always surrounded by macroscopic materials, such as the substrate, matrix, fiber probe, and so on. Previously [15,16], we proposed an effective interaction for such a nanometric system mediated by exciton-polaritons that exist in mixed states between photons and macroscopic material excitations instead of free photons. We showed that such a treatment provides a good description of the characteristics of an optical near field [17]. Using this, the electric displacement vector $\hat{D}(\mathbf{r})$ in Eq. (6) can be written as [18]

$$
\hat{D}(\mathbf{r}) = i \sqrt{\frac{2\pi}{V}} \sum_k \sum_{\lambda=1}^2 e_\lambda(k) f(k)(\hat{\xi}_k e^{i\mathbf{k} \cdot \mathbf{r}} - \hat{\xi}_k^\dagger e^{-i\mathbf{k} \cdot \mathbf{r}}),
$$

(9)

with

$$
f(k) = \frac{\hbar c k}{\sqrt{E(k)}} \sqrt{\frac{E^2(k) - E_m^2}{2E^2(k) - E_m^2 - \hbar^2 c^2 k^2}},
$$

(10)

where $\hbar$, $V$, $e_\lambda(k)$, and $\mathbf{k}$ are the Dirac constant, the quantization volume, the unit polarization vector, and the wavevector of the exciton-polaritons, respectively. Here we assume $e_\lambda(k)$ as real. The speed of light in a vacuum
is $c$, and the exciton-polariton energy with a wavevector $k$ and the macroscopic material excitation energy are $E(k)$ and $E_m$, respectively. Substituting Eqs. (7) and (9) into Eq. (6) gives the interaction Hamiltonian in the second-quantized representation as

$$\hat{H}_{\text{int}} = \sum_{\nu \nu' n' k \lambda} \left( \hat{c}^\dagger_{\nu n} \hat{c}_{\nu' n'} \xi_k g_{\nu \nu' n' k \lambda} - \hat{c}^\dagger_{\nu' n'} \hat{c}_{\nu n} \xi_k g_{\nu \nu' n' k \lambda} \right),$$

(11)

with

$$g_{\nu \nu' n' k \lambda} = -i \sqrt{2\pi} V f(k) \int \phi^*_{\nu n}(r)(\mu(r) \cdot e_{\lambda}(k)) e^{ik \cdot r} \phi_{\nu' n'}(r) dr.$$  

(12)

**Transition matrix element for exciton states**

In order to describe the creation and annihilation of excitons in a quantum dot, it is convenient to use the Wannier representation in which electrons are localized in an atomic site $R$. Then, the electron field operators can be expanded using the Wannier functions $w_{\nu R}(r)$ instead of $\phi_{\nu n}(r)$,

$$\psi(r) = \sum_{\nu = e,v} \sum_R \hat{c}_{\nu R} w_{\nu R}(r), \quad \psi^\dagger(r) = \sum_{\nu = e,v} \sum_R \hat{c}^\dagger_{\nu R} w^*_{\nu R}(r),$$

(13)

where $c_{\nu R}$ and $c_{\nu R}^\dagger$ denote the creation and annihilation operators of electrons at site $R$ in the energy band $\nu$. These operators in the Wannier representation are written in terms of $\hat{c}_{\nu n}$ and $\hat{c}^\dagger_{\nu n}$ in Eq. (7) as follows:

$$\hat{c}_{\nu R} = \sum_{\nu = e,v} \sum_n \hat{c}_{\nu n} \int w^*_{\nu R}(r) \phi_{\nu n}(r) dr,$$

(14a)

$$\hat{c}^\dagger_{\nu R} = \sum_{\nu = e,v} \sum_n \hat{c}^\dagger_{\nu n} \int w_{\nu R}(r) \phi^*_{\nu n}(r) dr.$$  

(14b)

When we assume excitons in the weak-confinement regime, i.e., an exciton Bohr radius to be smaller than the quantum-dot size, the exciton states in a quantum dot specified by the quantum number $m$ and $\mu$ can be described by superposition of the excitons in the Wannier representation as[19]

$$|\Phi_{m\mu}\rangle = \sum_{R, R'} F_m(R_{CM}) \varphi_\mu(\beta) \hat{c}^\dagger_{\nu R} \hat{c}_{\nu R'} |\Phi_g\rangle$$

$$= \sum_{R, R'} F_m(R_{CM}) \varphi_\mu(\beta) \sum_{n, n'} h_{R n R' n'} \hat{c}^\dagger_{c n} \hat{c}_{c n'} |\Phi_g\rangle,$$

(15)

where $F_m(R_{CM})$ and $\varphi_\mu(\beta)$ denote the envelope functions for the center of mass and relative motions of the excitons, respectively. These are $R_{CM} = (m_c R' + m_h R)/(m_c + m_h)$ and $\beta = R' - R$, where $m_c$ and $m_h$ are the
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The overlap integrals \( h_{RnR'n'} \) are defined as

\[
h_{RnR'n'} = \int \int w_n^*(r_2)w_{cR'}(r_1)\phi_{cn}(r_1)\phi_{vn'}(r_2)dr_1dr_2.
\]

The sum of \( \nu' \) in Eq. (14) is determined automatically as \( \hat{c}_{cn}^\dagger \) and \( \hat{c}_{vn} \), because the valence band is fully occupied in the initial ground state \( |\Phi_g\rangle \). Using Eqs. (11) and (15), the transition matrix element from the exciton state to the ground state is obtained as

\[
\langle \Phi_g | \hat{H}_{\text{int}} | \Phi_{m\mu} \rangle = \sum_{n_1, n_2} \sum_{R, R'} F_m(R_{CM}) \varphi_\mu(\beta)
\]

\[
\times \sum_k \sum_{\lambda=1}^2 (\xi_k g_{vn_1cn_2\lambda} - \xi_k^\dagger g_{vn_1cn_2-\lambda})h_{Rn_1R'n_2}.
\]

where we use the following relation:

\[
\langle \Phi_g | \hat{c}_{vn_1}^\dagger \hat{c}_{cn_2} \hat{c}_{vn_4}^\dagger \hat{c}_{vn_3} | \Phi_g \rangle = \delta_{n_1, n_4} \delta_{n_2, n_3}.
\]

In addition, with the help of the completeness and orthonormalization of \( \phi_{vn}(r) \) [see Eq. (8)], we can simplify the product of \( g \) and \( h \) as

\[
\sum_{n_1, n_2} g_{vn_1cn_2\lambda} h_{Rn_1R'n_2} = -i \sqrt{\frac{2\pi}{V}} f(k) \int w_n^*(r)\mu(r)w_{cR'}(r) \cdot e_\lambda(k)e^{ik \cdot r} dr \\
\approx -i \sqrt{\frac{2\pi}{V}} f(k)(\mu_{cv} \cdot e_\lambda(k))e^{ik \cdot R} \delta_{R, R'},
\]

where the transformation of the spatial integral in the first line of Eq. (19) into the sum of the unit cells and the spatial localization of the Wannier functions provides \( \delta_{R, R'} \) in the second line. The transition dipole moment for each unit cell is defined as

\[
\mu_{cv} = \int_{\text{UC}} w_n^*(r)\mu(r)w_{cR}(r) dr.
\]

We assume that the transition dipole moment is the same as that of the bulk material, independent of the site \( R \), and that the electric displacement vector is uniform at each site. Finally, Eq. (17) is reduced to

\[
\langle \Phi_g | \hat{H}_{\text{int}} | \Phi_{m\mu} \rangle = -i \sqrt{\frac{2\pi}{V}} \sum_R \sum_k \sum_{\lambda=1}^2 f(k)(\mu_{cv} \cdot e_\lambda(k))F_m(R)\varphi_\mu(0)
\]

\[
\times (\xi_k e^{ik \cdot R} - \xi_k^\dagger e^{-ik \cdot R}).
\]

Here, we note that the exciton-polariton field expanded by the plane wave with the wavevector \( k \) depends on the site \( R \) in the quantum dot because we do not apply the long wave approximation that is usually used for far-field light.
2.3 Optical Near-Field Coupling Between Quantum Dots

Formulation

Until now, we have derived the transition matrix element from the exciton state to the ground state in a quantum dot. Considering operations of nanophotonic devices, signal carrier corresponds to the energy transfer between nanometric objects, or quantum dots, which are electronically separated, and the speed of the energy transfer is determined by the coupling strength of an optical near field. In this stage, we derive the coupling strength,

\[ \hbar U = \langle \Psi_f | \hat{H}_{\text{int}} | \Psi_i \rangle, \]  

(22)

where \(|\Psi_i\rangle\) and \(|\Psi_f\rangle\) represent exact initial and final states, respectively, in which the states consist of quantum-dot states, photon fields, and some external degrees of freedom, such as a substrate and a glass fiber probe. Since the exact states can not be given rigorously, we deal with the problem for taking the minimum matter and photon states by using the projection operator method, where the theoretical treatment in such complex system comes down to two body problem as we have reported in detail[16,20].

We can rewrite the exact eigenstate as two substates which belong in a relevant P-space and an irrelevant Q-space, which are expressed by using projection operators \(P\) and \(Q\) as \(|\Psi^P\rangle = P|\Psi\rangle\) and \(|\Psi^Q\rangle = Q|\Psi\rangle\), respectively, where \(\lambda = i, f\). Here, \(P\) and \(Q\) are specified by the following relations: \(P + Q = 1\), \(P^2 = P\), \(Q^2 = Q\), \(P^\dagger = P\), and \(Q^\dagger = Q\)\[^2\]. In the case of two-quantum-dot system, P-space is constructed from the eigenstates of \(\hat{H}_0\), i.e., a combination of the two energy levels for each quantum dot and the exciton-polariton vacuum state. In Q-space, which is complementary to P-space, the exciton-polariton states are not vacant. According to this notation, the exact state can be formally expressed by using the state in P-space only as

\[ |\Psi_\lambda\rangle = \hat{J}P(P\hat{J}^\dagger P)^{-1/2}|\Psi^P_\lambda\rangle, \]  

(23)

where

\[ \hat{J} = \left[ 1 - (E_\lambda - \hat{H}_0)^{-1}Q\hat{H}_{\text{int}} \right]^{-1}, \]  

(24)

and \(E_\lambda\) represents the eigenenergy of the total Hamiltonian \(\hat{H}\). Using Eq. (23), we can obtain the effective interaction \(\hat{H}_{\text{eff}}\) as

\[ \langle \Psi_f | \hat{H}_{\text{int}} | \Psi_i \rangle = \langle \Psi^P_f | \hat{H}_{\text{eff}} | \Psi^P_i \rangle, \]  

(25)

where

\[ \hat{H}_{\text{eff}} = (P\hat{J}^\dagger P)^{-1/2}(P\hat{J}^\dagger \hat{H}_{\text{int}} \hat{J} P)(P\hat{J}^\dagger P)^{-1/2}. \]  

(26)

To further evaluate Eq. (25), we approximate operator \(\hat{J}\) and eigenvalue \(E_\lambda\) perturbatively with respect to \(\hat{H}_{\text{int}}\); that is, \(\hat{J} = 1 + (E^P_0 - E^Q_0)^{-1}\hat{H}_{\text{int}} + \cdots\).
Since the lowest order is $\langle \Psi_i^P | \hat{H}_{\text{int}} | \Psi_i^P \rangle = 0$, Eq. (25) is rewritten within the second order as

$$hU = \sum_m \langle \Psi_i^P | \hat{H}_{\text{int}} | m^Q \rangle \langle m^Q | \hat{H}_{\text{int}} | \Psi_i^P \rangle \left( \frac{1}{E_{0i}^P - E_{0m}^Q} + \frac{1}{E_{0f}^P - E_{0m}^Q} \right), \quad (27)$$

where $E_{0i}^P$, $E_{0f}^P$, and $E_{0m}^Q$ represent the eigenenergies of the unperturbed Hamiltonian for the initial and final states in P-space and the intermediate state in Q-space, respectively. Since we focus on the interdot interaction of Eq. (27), we set the initial and final states in P-space to $| \Psi_i^P \rangle = | \Phi_A^m \rangle | \Phi_B^g \rangle | 0 \rangle$ and $| \Psi_f^P \rangle = | \Phi_A^g \rangle | \Phi_B^m \rangle | 0 \rangle$. Then, the intermediate states in Q-space that involve exciton-polaritons with the wavevector $k$ are utilized for the energy transfer from one quantum dot to the other, according to $| m^Q \rangle = | \Phi_A^g \rangle | \Phi_B^g \rangle | k \rangle$ and $| \Phi_A^m \rangle | \Phi_B^{m'} \rangle | k \rangle$. The superscripts $A$ and $B$ are used to label two quantum dots. Substituting Eq. (21), one can rewrite Eq. (27) as

$$hU = \varphi_A^m(0) \varphi_B^{m'}(0) \int \int F_m^A(R_A) F_{m'}^B(R_B)$$

$$\times (Y_A(R_B - R_A) + Y_B(R_A - R_B)) dR_A dR_B, \quad (28)$$

where the sum of $R_\alpha$ ($\alpha = A, B$) in Eq. (21) is transformed to the integral form. The functions $Y_\alpha(R_{AB})$, which connect the two spatially isolated two envelope functions $F_m^A(R_A)$ and $F_m^B(R_B)$, are defined as

$$Y_\alpha(R_{AB}) = -\frac{1}{4\pi^2} \sum_{\lambda=1}^2 \int (\mu_{\lambda}^A \cdot \hat{e}_\lambda(k))(\mu_{\lambda}^B \cdot \hat{e}_\lambda(k)) f^2(k)$$

$$\times \left( \frac{e^{ik_1 \cdot R_{AB}}}{E(k) + E_\alpha} + \frac{e^{-ik_1 \cdot R_{AB}}}{E(k) - E_\alpha} \right) dk, \quad (29)$$

where $R_{AB} = R_A - R_B$ is used.

In order to obtain an explicit functional form of $Y_\alpha(R_{AB})$, we apply the effective mass approximation to the exciton-polaritons,

$$E(k) = \frac{\hbar^2 k^2}{2m_p} + E_m, \quad (30)$$

where $m_p$ is the exciton-polariton effective mass. Using this approximation, Eq. (29) can be transformed into

$$Y_\alpha(R_{AB}) =$$

$$\left( \mu_{\lambda}^A \cdot \mu_{\lambda}^B \right) \left[ W_+ e^{-\Delta_\alpha + R_{AB}} \left( \frac{\Delta_\alpha^2 + \Delta_{\alpha - R_{AB}}}{R_{AB}^2} + \frac{1}{R_{AB}^3} \right)$$

$$- W_+ e^{-\Delta_\alpha - R_{AB}} \left( \frac{\Delta_\alpha^2 - \Delta_{\alpha + R_{AB}}}{R_{AB}^2} + \frac{1}{R_{AB}^3} \right) \right]$$
\[-(\mu_c^A \cdot \hat{R}_{AB})(\mu_c^B \cdot \hat{R}_{AB})\left[ W_{\alpha+} e^{-\Delta_{\alpha+} R_{AB}} \left( \frac{\Delta_{\alpha+}^2}{R_{AB}^2} + \frac{3 \Delta_{\alpha+}}{R_{AB}^3} + \frac{3}{R_{AB}} \right) \right. \]

\[- W_{\alpha-} e^{-\Delta_{\alpha-} R_{AB}} \left( \frac{\Delta_{\alpha-}^2}{R_{AB}^2} + \frac{3 \Delta_{\alpha-}}{R_{AB}^3} + \frac{3}{R_{AB}} \right) \right], \quad (31)\]

where $R_{AB}$ and $\hat{R}_{AB}$ are the absolute value $|R_{AB}|$ and the unit vector defined by $R_{AB}/|R_{AB}|$, respectively. The weight coefficients $W_{\alpha\pm}$ and decay constants $\Delta_{\alpha\pm}$ are given by

\[ W_{\alpha\pm} = \frac{E_p}{E_\alpha} \frac{(E_m - E_\alpha)(E_m + E_\alpha)}{(E_m - E_p \mp E_\alpha)(E_m \pm E_\alpha) - E_m^2/2}, \quad (32a) \]

\[ \Delta_{\alpha\pm} = \frac{1}{\bar{\hbar} c} \sqrt{E_p(E_m \pm E_\alpha)}, \quad (32b) \]

where the exciton-polariton effective mass is rewritten as $E_p = m_p c^2$. Since the dipole moments $\mu_c^A$ and $\mu_c^B$ are not determined as fixed values, we assume that they are parallel, and take a rotational average of Eq. (31). Therefore, $\langle (\mu_c^A \cdot \hat{R}_{AB})(\mu_c^B \cdot \hat{R}_{AB}) \rangle = \mu_{c\alpha}^A \mu_{c\alpha}^B / 3$ with $\mu_{c\alpha} = |\mu_{c\alpha}|$, and we obtain the final form of the function $Y_{\alpha}(R_{AB})$ as

\[ Y_{\alpha}(R_{AB}) = \frac{2 \mu_{c\alpha}^A \mu_{c\alpha}^B}{3 R_{AB}^2} (W_{\alpha+} e^{-\Delta_{\alpha+} R_{AB}} - W_{\alpha-} e^{-\Delta_{\alpha-} R_{AB}}). \quad (33) \]

Equation (33) is the sum of two Yukawa functions with a short and long interaction range (heavy and light effective mass) given in Eq. (32b). We can estimate the coupling strength between two quantum dots from the analytic form of the interaction potential given by Eqs. (28) and (33), and we can show the existence of dipole-forbidden energy transfer driven by the optical near-field coupling, as discussed in the following.

**Numerical results**

In this subsection, we give typical values of the coupling strength of $\hbar U$ in Eq. (28) using an example of CuCl quantum cubes embedded in a NaCl matrix. Due to the effect of size confinement, the center of mass motion and relative motion for an exciton in a CuCl quantum cube are\[^{[19]}\]

\[ F_{m}^{\alpha}(R_{a}) = \left( \frac{2}{L_{\alpha}} \right)^{3/2} \sin \left( \frac{\pi m_{x} x_{\alpha}}{L_{\alpha}} \right) \sin \left( \frac{\pi m_{y} y_{\alpha}}{L_{\alpha}} \right) \sin \left( \frac{\pi m_{z} z_{\alpha}}{L_{\alpha}} \right), \quad (34a) \]

\[ \varphi_{1s}(r) = \frac{1}{\sqrt{\pi a^3}} e^{-r/a}, \quad (34b) \]

respectively, where the atomic site and the quantum number are represented by $R_{a} = (x_{\alpha}, y_{\alpha}, z_{\alpha})$ with $\alpha = A, B$ and $m = (m_{x}, m_{y}, m_{z})$ with $m_{x}, m_{y}, m_{z} = 1, 2, 3, \cdots$. The variables $L_{\alpha}$ and $a$ denote a width of the quantum cube and the Bohr radius of the exciton, respectively. Here, we assume relative motion
in the $1s$ state. The coupling strength is obtained numerically by substituting Eqs. (33) and (34) into Eq. (28). In Fig. 5(a), the calculation results are plotted as a function of the intercube distance. The curve with square dots represents the coupling between the dipole-active exciton levels, i.e., $m = m' = (1,1,1)$, in two quantum cubes. When we set the intercube distance and a width of the quantum cubes as $d = 5$ nm and $L_A = L_B = 10$ nm, respectively, the coupling strength is about 89 $\mu$eV ($U^{-1} = 7.4$ ps). The curve with circular dots is the result for $m = (1,1,1)$ and $m' = (2,1,1)$. For the conventional far-field light, $m' = (2,1,1)$ is the dipole-inactive exciton level, and it follows that the optical near-field interaction inherently involves such a transition because of the finite interaction range. Fig. 5(b) is a schematic illustration of the dipole-inactive transition, in which the optical near field enables to excite the local dipoles at the near side in a quantum dot with dipole-active level for the far-field light. This coupling strength is estimated from Fig. 5(a) as $\hbar U = 37$ $\mu$eV ($U^{-1} = 17.7$ ps) for $d = 5$ nm, and $\hbar U = 14$ $\mu$eV ($U^{-1} = 46.9$ ps) for $d = 15$ nm, where the cube sizes are set as $L_A = 10$ nm and $L_B = 14.1$ nm to realize resonant energy transfer between the exciton state in QD-A and the first exciton excitation state in QD-B. The coupling strength ($m \neq m'$) is approximately half that of $m = m'$ at the same intercube distance, but it is strong enough for our proposed nanophotonic devices. For functional operations, the difference between the coupling strengths is important to divide the system into two parts, i.e., a quantum mechanical part and a classical dissipative part, as already illustrated in Fig. 4.

2.4 Summary

In this section, we formulate a optical near-field coupling by using appropriate bases which are constructed form typical excitonic states in a quantum dot and exciton-polariton state in a surrounding system, and not using the long wave approximation which often applies to a conventional optical interaction in an atomic system. Although we have derived the coupling in the lowest order as given in Eq. (27), our formulation would be exact if we take rigorous eigenstate of exciton-polaritons as the intermediate states, instead of the simple effective mass approximation which is applied in the above discussion. However, in the following sections, our interests are characteristic functional operations of nanophotonic devices on the basis of certain coupling strength of the optical near field, rather than to understand fundamental properties of optical near-field coupling. More rigorous description of the optical near-field coupling will discuss elsewhere.

From numerical results shown in Fig. 5, the coupling strength of optical near field depends on the interdot distance, which is one of key features for nanophotonic device operations. By using this, we can control the dynamics of energy flow in a nanometric space and develop some functional operations inherent in nanophotonic devices. Furthermore, we showed that dipole inactive
energy transfer can occur when a distance between isolated quantum systems becomes enough small, which is related to the energy states in nanometric objects as well as steeply gradient spatial distribution of the optical near field. Especially, the dipole-inactive energy transfer between the states with different quantum numbers enables to realize unidirectional energy transfer in a nanometric system with the help of fast relaxation of exciton sublevels. This is a quite important feature for signal isolation in nanophotonic devices. In Secs. 3 and 4, we discuss operation principles of various functional devices by using such features of the optical near-field coupling skillfully.

3 Nanophotonic Switch Based on Dissipation Control

In the previous section, we had theoretically explained that an exciton in a dipole-inactive energy level can be excited by using an optical near field. A relaxation time of the exciton in the dipole-inactive level, the higher energy sublevel, is generally in the order of a few ps because of the strong coupling between an exciton and a phonon reservoir in a surrounding system[22]. Since the coupling strength of the optical near field corresponds to about subhundred ps, which has been estimated in Sec. 2, the intra-sublevel relaxation is as a figure fast as in the order of energy transfer between two quantum dots. Therefore, unidirectional energy transfer can be realized in a
two or more quantum-dot system by mediating the intra-sublevel relaxation. On the other hand, we can create and annihilate an exciton in an exciton ground state by using external pumping light. Excitons in a quantum-dot system affect exciton-exciton interaction in a quantum dot, because more than an exciton confined in a nanometric space. We have qualitatively regarded the excitons as fermionic particles, that is of course exact. When the lowest energy sublevel is occupied, the exciton population cannot drop into the lowest energy level, and thus, we can change the dissipation path selectively by arranging several quantum dots. This selectivity reads the origin of a nanophotonic switching operation.

In this section, we investigate our proposed nanophotonic switch, which is a basic element of nanophotonic devices[23]. Figure 6 illustrates a switch that consists of three quantum dots (cubes) with discrete exciton energy levels depending on the quantum-dot size. The one-side lengths of these cubes are chosen in the ratio $1 : \sqrt{2} : 2$, so that the adjacent quantum dots have resonant energy levels. The principle of operation of the switch is as follows: as shown in Fig. 6(a), an exciton or population is created at the $(1,1,1)$-level in QD-I as an initial condition. Then the population is transferred to QD-O and QD-C as a result of an optical near-field coupling. Owing to the fast relaxation between sublevels in each dot via exciton-phonon coupling, the population is transferred to lower energy levels, and finally collected at the lowest $(1,1,1)$-level in QD-C. This corresponds to the OFF-state of the switch, and, consequently, we obtain no output signals from the output port, i.e., the $(1,1,1)$-level in QD-O. By contrast, in the ON-state of the switch [Fig. 6(b)], the $(1,1,1)$-level in QD-C is initially filled by the control light, isolating QD-C from the other two quantum dots. The input population only reaches the $(1,1,1)$-level in QD-O and can be detected as output signals, either by the optical near-field coupling to the detector or by the far-field light emitted with electron-hole recombination.

From the above explanation, we understand that the key parameters determining the response time of the device are the coupling strength between two quantum dots via an optical near fields, and that between excitons and a phonon reservoir. In Sec. 3.1, dynamics of exciton population is formulated on the basis of quantum mechanical density matrix formalism, where we consider the phonon field as well as the optical near field discussed in Sec. 2, and roles of some key parameters in such a quantum-dot system are numerically clarified. This allows us to discuss the temporal dynamics of our proposed nanophotonic. We evaluate the response time of the CuCl quantum-cube system as a numerical example, which have been extensively examined in experimental and theoretical studies of quantum dots[19,22,24,25]. Section 3.2 devotes to evaluate switching operations in a three quantum-dot system as shown in Fig. 6, where the effect of state-filling is introduced phenomenologically. Furthermore, faster iterative switching operations can be achieved in the order of 100 ps, when we apply appropriate control light pulse for uti-
lizing stimulated absorption and emission effectively, which will be discussed by means of numerical analysis in Sec. 3.3.

3.1 Dynamics in a Two-Quantum-Dot System with Dissipation

As mentioned above, relaxation in the exciton sublevels guarantees the unidirectional energy transfer in a system with several quantum dots. The relaxation originates from coupling between exciton excited state and lattice vibrations in a quantum dot and surrounding matter which are regarded as a phonon reservoir. In order to understand energy transfer dynamics in such a quantum-dot system, which goes through a dissipative process, we first examine a two-quantum-dot system coupled to the phonon reservoir.

Formulation

In Fig. 7, we schematically illustrate a considered two-quantum-dot system and a phonon reservoir system, in which all energy transfer paths are depicted except for the coupling to the far-field light because of different time scales. The Hamiltonian of the system is modelled as

\[ \hat{H} = \hat{H}_0 + \hat{H}_{\text{int}} + \hat{H}_{\text{SR}}, \]

and

\[ \hat{H}_0 = \hbar \Omega_2 \hat{A}^\dagger \hat{A} + \hbar \Omega_1 \hat{B}_1^\dagger \hat{B}_1 + \hbar \Omega_2 \hat{B}_2^\dagger \hat{B}_2 + \hbar \sum_n \omega_n \hat{b}_n^\dagger \hat{b}_n, \] (35a)

\[ \hat{H}_{\text{int}} = \hbar U (\hat{A}^\dagger \hat{B}_2 + \hat{B}_2^\dagger \hat{A}), \] (35b)

\[ \hat{H}_{\text{SR}} = \hbar \sum_n (g_n \hat{b}_n^\dagger \hat{B}_1^\dagger \hat{B}_2 + g_n^* \hat{b}_n \hat{B}_2^\dagger \hat{B}_1). \] (35c)
When we assume that initial and final states are constructed only in terms of one-exciton states, the creation (annihilation) operators of excitons can be written as follows: \( \hat{A}\dagger = |e\rangle\langle g|_A \) (\( \hat{A} = |g\rangle\langle e|_A \)), \( \hat{B}_{1}\dagger = |e\rangle\langle g|_B_1 \), (\( \hat{B}_1 = |g\rangle\langle e|_B_1 \)), and \( \hat{B}_2\dagger = |e\rangle\langle g|_B_2 \), (\( \hat{B}_2 = |g\rangle\langle e|_B_2 \)). We can readily understand the following commutation relations: \[ [\hat{B}_{1}\dagger, \hat{B}_j] = \delta_{i,j}(|e\rangle\langle e|_B_i - |g\rangle\langle g|_B_i) \]
and \[ [\hat{B}_i, \hat{B}_{j}\dagger] = [\hat{B}_{i}\dagger, \hat{B}_j] = 0 \] (\( i, j = 1, 2 \)). Therefore, the operators are neither bosonic nor fermionic. Bosonic operators \( (\hat{b}_{n}\dagger, \hat{b}_n) \) are for the phonons with eigenenergy \( \hbar\omega_n \). For simplicity, the rotating wave approximation is used in the interaction Hamiltonian \( \hat{H}_\text{int} \) as \( (\hat{A} + \hat{A}\dagger)\hat{B}_2 + \hat{B}_2\dagger \approx \hat{A}\dagger \hat{B}_2 + \hat{A}\hat{B}_2\dagger \). Phonon reservoir is assumed to be a collection of multiple harmonic oscillators labelled \( n \). Note that the exciton-polariton degrees of freedom have already been traced out, and thus the coupling strength of the optical near field, \( \hbar U \), appears in Eq. (36b).

Dynamics of an exciton in this system is given by the following Liouville equation [26,27]

\[
\dot{\hat{\rho}}(t) = -\frac{i}{\hbar} [\hat{H}, \hat{\rho}(t)], \tag{37}
\]
where \( \hat{\rho}(t) \) represents the density operator, traced out the exciton-polariton degrees of freedom. In order to express the second-order temporal correlation clearly, the formal solution of Eq. (37) in the integral form is again substituted into the right hand side of Eq. (37), and thus,

\[
\hat{\rho}^I(t) = -\frac{i}{\hbar} [\hat{H}_\text{int} + \hat{H}_\text{SR}^I(t), \hat{\rho}^I(0)]
- \frac{1}{\hbar^2} \int_0^t [\hat{H}_\text{int} + \hat{H}_\text{SR}^I(t'), [\hat{H}_\text{int} + \hat{H}_\text{SR}^I(t'), \hat{\rho}^I(t')]] dt', \tag{38}
\]
where the upperscript \( I \) means the interaction picture, and the relation \( \hat{H}_\text{int}^I(t) = \hat{H}_\text{int} \) is used [26]. Since we are interested in the exciton population in the two-quantum-dot system, we take a trace with respect to the degrees of freedom of the phonon reservoir as \( \hat{\rho}_S^I(t) = \text{Tr}_R[\hat{\rho}_S^I(t')] \). Here, the density operator is assumed to be a direct product of the quantum-dot system part \( \hat{\rho}_S^I(t) \) and the reservoir system part \( \hat{\rho}_R^I(t) \). If the reservoir has a very large volume, deviation from the initial value can be neglected, and the density

\[ QD-A \quad QD-B \]
operator is approximated as
\[
\hat{\rho}'(t) = \hat{\rho}'_S(t)\hat{\rho}'_R(t) \approx \hat{\rho}'_S(t)\hat{\rho}_R(0),
\] (39)

which corresponds to the Born approximation[26]. Taking a trace on both sides of Eq. (38) about the reservoir operator, we obtain
\[
\dot{\hat{\rho}}_S(t) = -iU(r)[\hat{A}^\dagger \hat{B}_2 + \hat{B}_2^\dagger \hat{A}]
- \sum_n n(\omega_n, T)[\{\hat{C}^\dagger \hat{C}, \hat{\rho}'_S(t)\} - 2\hat{C}^\dagger \hat{\rho}'_S(t)\hat{C}] \otimes \gamma_n(t)
- i[\hat{C}^\dagger \hat{C}, \hat{\rho}'_S(t)] \otimes \gamma_n(t)
- \sum_n [1 + n(\omega_n, T)][\{\hat{C}^\dagger \hat{C}, \hat{\rho}'_S(t)\} - 2\hat{C}^\dagger \hat{\rho}_S(t)\hat{C}^\dagger] \otimes \gamma_n(t)
+ i[\hat{C}^\dagger \hat{C}, \hat{\rho}'_S(t)] \otimes \gamma_n(t),
\] (40)

where the curly brackets \{\} represent the anti-commutation relation, and the notation \(\otimes\) designates the convolution integral. In order to avoid verbose expression, we make the following replacement; \(\hat{C}^\dagger = \hat{B}_2^\dagger \hat{B}_1\) and \(\hat{C} = \hat{B}_1^\dagger \hat{B}_2\).

Since we assume that the reservoir system is at equilibrium, the terms including \(\text{Tr}_R[\hat{b}_n^\dagger \hat{\rho}_R(0)]\) and \(\text{Tr}_R[\hat{b}_n \hat{\rho}_R(0)]\) disappear in Eq. (40). The number of phonons in the equilibrium state is written as \(n(\omega_n, T) = \text{Tr}_R[\hat{b}_n^\dagger \hat{b}_n \hat{\rho}_R(0)]\), and it follows Bose-Einstein statistics as
\[
n(\omega_n, T) = \frac{1}{e^{\hbar \omega_n/k_B T} - 1}.
\] (41)

The real and imaginary parts of function
\[
\gamma_n(t) = |g_n|^2 e^{i(\Delta \omega - \omega_n)t}
\] (42)
with \(\hbar \omega = \hbar(\Omega_2 - \Omega_1)\) are represented as \(\gamma_n^r(t)\) and \(\gamma_n^i(t)\), respectively, and are related to the relaxation (real part) and energy shift (imaginary part) of the energy level in QD-B that is originated from the coupling to the phonon reservoir. The convolution integral in Eq. (40) expresses a memory effect due to time delay in the phonon reservoir. However, if the dynamics of the reservoir system are much faster than those of the two-quantum-dot system, one can approximate the density operator of the two-dot system as \(\hat{\rho}'_S(t-t') = \hat{\rho}'_S(t)\) (a Markov approximation). Using this approximation, and rewriting the summation as \(\sum_n = \int_0^\infty D(\omega) d\omega\), with \(D(\omega)\) being the density of states for each phonon, we can express the convolution integral analytically as
\[
\sum_n n(\omega_n, T)\hat{\rho}'_S(t) \otimes \gamma_n(t)
= \hat{\rho}'_S(t) \int_0^\infty n(\omega, T)D(\omega)|g(\omega)|^2 \left(\int_0^t e^{i(\Delta \omega - \omega)t'} dt'\right) d\omega.
\]
\[ \approx \hat{\rho}_S^I(t) \left[ \pi n(\Delta \omega, T) D(\Delta \omega) |g(\Delta \omega)|^2 + iP \int_0^\infty \frac{n(\omega, T) D(\omega) |g(\omega)|^2}{\Delta \omega - \omega} d\omega \right]. \]  

(43)

Here, we extend the upper limit of the time integration to infinity. The equation of motion for the dot system is finally reduced to

\[ \dot{\hat{\rho}}_S^I = iU(r) [\hat{A}^\dag \hat{B}_2 + \hat{B}_2^\dag \hat{A}, \hat{\rho}_S^I(t)] - n \gamma \{\hat{C} \hat{C}^\dag, \hat{\rho}_S^I(t)\} - 2 \hat{C}^\dag \hat{\rho}_S^I(t) \hat{C}, \]  

(44)

where \( n = n(\Delta \omega, T) \) and \( \gamma = \pi D(\Delta \omega) |g(\Delta \omega)|^2 \). The terms indicating the energy shift are neglected in Eq. (44) because the shift is usually small in the case of weak coupling between the quantum-dot system and phonon reservoir.

Let us consider one-exciton dynamics in the system, using three bases, as illustrated in Fig. 8. The equations of motion for the matrix elements are then read in the Schrödinger picture as

\[
\begin{align*}
\dot{\rho}_{11}(t) &= iU(r) [\rho_{12}(t) - \rho_{21}(t)], \\
\dot{\rho}_{12}(t) - \dot{\rho}_{21}(t) &= 2iU(r) [\rho_{11}(t) - \rho_{22}(t)] - (1 + n) \gamma [\rho_{12}(t) - \rho_{21}(t)], \\
\dot{\rho}_{22}(t) &= -iU(r) [\rho_{12}(t) - \rho_{21}(t)] - 2(1 + n) \gamma \rho_{22}(t) + 2n \gamma \rho_{33}(t), \\
\dot{\rho}_{33}(t) &= 2(1 + n) \gamma \rho_{22}(t) - 2n \gamma \rho_{33}(t),
\end{align*}
\]  

(45a-d)

where \( \rho_{mn}(t) \equiv \langle \Phi_m | \hat{\rho}_S(t) | \Phi_n \rangle \) is employed. When the temperature, \( T \), equals zero (\( n = 0 \)), Eq. (45) can be solved analytically. The diagonal parts representing the population probability for each energy level in QD-A and QD-B, as well as the off-diagonal parts representing quantum coherence, are given as

Fig. 8. Three bases of the single-exciton state in a two-quantum-dot system.
\[ \rho_{11}(t) = \frac{1}{Z^2} e^{-\gamma t} \left[ \frac{\gamma}{2} \sinh(Zt) + Z \cosh(Zt) \right]^2, \quad (46a) \]
\[ \rho_{22}(t) = \frac{U^2}{Z^2} e^{-\gamma t} \sinh^2(zt), \quad (46b) \]
\[ \rho_{33}(t) = 1 - [\rho_{11}(t) + \rho_{22}(t)], \quad (46c) \]
\[ \rho_{12}(t) = -\rho_{21}(t) = \frac{U}{Z^2} e^{-\gamma t} \sinh(Zt) \left[ \frac{\gamma}{2} \sinh(Zt) + Z \cosh(Zt) \right], \quad (46d) \]

where \( Z \equiv \sqrt{(\gamma^2/2) - U^2} \), and initial conditions \( \rho_{11}(0) = 1 \) and \( \rho_{22}(0) = \rho_{33}(0) = 0 \) are used. We define the state-filling time \( \tau_S \) as \( \rho_{33}(\tau_S) = 1 - e^{-1} \), which corresponds to the time for the excitation energy transfer from QD-A to the lower energy level in QD-B. From Eq. (46), it follows that the temporal evolution of the population is quite different at \( U > \gamma/2 \) and \( U < \gamma/2 \). Although Eq. (46) seems to be undefined at \( U = \gamma/2 \) (\( Z = 0 \)), taking a limit value, there is a definite solution regardless of whether \( Z \to +0 \) or \( -0 \) is taken. In Fig. 9, the state-filling time \( \tau_S \) is plotted as a function of the ratio of \( \gamma/2 \) to \( U \). For \( U > \gamma/2 \), population shows damped oscillation with envelope function \( e^{-\gamma t} \); thus, \( \tau_S \) is determined by the relaxation constant \( \gamma \), i.e., \( \tau_S \sim \gamma^{-1} \). By contrast, for \( U < \gamma/2 \), \( \rho_{22}(t) \) decays monotonically. At first glance, as \( \gamma/2 \) increases, we expect \( \tau_S \) to decrease monotonically, because the population flows into the lower energy level more quickly; nevertheless, \( \tau_S \) increases again, as shown in Fig. 9. This occurs because the upper energy level in QD-B becomes effectively broad with increasing \( \gamma \), which results in a decrease in the resonant energy transfer between the quantum dots. When the ratio \( \gamma/2U \) is sufficiently large, \( \tau_S \) increases linearly, as seen in Fig. 9. Therefore, the state-filling time is not only determined by the coupling strength between two quantum dots via the optical near field, but also by the coupling strength to the phonon reservoir system. From Fig. 9, it follows that the fastest energy transfer is obtained when \( \gamma/2 \sim U \) is satisfied.

The term \( 2\gamma n \rho_{33}(t) \) on the right-hand side of Eq. (45c) indicates that the finite temperature effect due to the finite number of phonons \( n \neq 0 \) induces back transfer of the excitation energy from the reservoir to the two-quantum-dot system. Within the Born approximation adopted in Eq. (39), this term incoherently increases population \( \rho_{22}(t) \). As population \( \rho_{33}(t) \) increases, the back transfer becomes large, and gives residual populations \( \rho_{11}(t) \) and \( \rho_{22}(t) \) in the upper-levels in both quantum dots.

**Numerical results**

Using Eqs. (45) and (46), we present numerical results for the dynamics of exciton population for zero and finite temperatures in order to verify the theoretical consideration just presented, and to estimate the state-filling time for some practical cases. Suppose that the system consists of two CuCl quantum cubes embedded in a NaCl matrix. The quantum cube size is set as \( a : \sqrt{2a} \),
so that the (1,1,1)-level of QD-A is resonant with the (2,1,1)-level of QD-B. The variable parameters are the coupling strength $\hbar U$ or the intercube distance, the relaxation constant $\gamma$, and the temperature $T$ of the phonon reservoir system.

In Figs. 10(a) and (b), the $E_2$ and $E_1$-level populations $\rho_{22}(t)$ and $\rho_{33}(t)$ in QD-B are plotted for the coupling strengths of the optical near field, $\hbar U = 100, 60, \text{ and } 40 \mu eV$ ($U = 1.51 \times 10^{-1}, 0.90 \times 10^{-1}, \text{ and } 0.60 \times 10^{-1} \text{ ps}^{-1}$). In order to investigate a behavior of the dynamics around the critical condition $U = \gamma/2$, appropriate coupling strengths are chosen, though these are a little strong compared with the estimated values in Sec. 2. The relaxation constant $\gamma$ is assumed to be $(1 \text{ ps})^{-1}$ from the experimental study[22], and the temperature is $T = 0 \text{ K}$. The exciton populations for all three cases in Fig. 10(a) show monotonic decays, because $U < \gamma/2$ is satisfied. From Fig. 10(b), the state-filling time $\tau_S$ is estimated for the coupling strength $\hbar U = 100, 60, \text{ and } 40 \mu eV$ as 22, 60, and 140 ps, respectively. This indicates that an energy transfer time of less than 100 ps can be realized when the coupling strength is larger than about 50 $\mu eV$.

Figure 11 shows the result for $\gamma^{-1} = 10 \text{ ps}$, which corresponds to weaker coupling between an exciton and the phonon reservoir. Damped oscillation due to mutation between two resonant levels is clearly seen for $\hbar U = 100 \mu eV$ in Fig. 11(a), where $U > \gamma/2$ is satisfied. Although $U > \gamma/2$ is also satisfied for $\hbar U = 60 \text{ and } 40 \mu eV$, we cannot observe the oscillation because of the small amplitudes. From Fig. 11(b), the state-filling time $\tau_S$ for the three cases is estimated as 12, 18, and 25 ps, respectively. Compared with Fig. 10(b), the state-filling speed becomes faster in Fig. 11(b) in spite of the decrease in $\gamma$ because $U$ is nearly equal to $\gamma/2$. Figures 10 and 11 indicate that the intercube distance should be adjusted so that the optical near-field coupling.
Fig. 10. Temporal-evolution of (a) $E_1$-level population $\rho_{22}(t)$ and (b) $E_2$-level population $\rho_{33}(t)$, where the relaxation constant $\gamma$ and temperature $T$ are set as $(1 \text{ ps})^{-1}$ and 0 K, respectively. The solid, dashed, and dotted curves represent the cases for coupling strengths, $\hbar U = 100 \mu\text{eV}$, 60, and 40 $\mu\text{eV}$, respectively.

Fig. 11. Temporal-evolution of (a) $E_1$-level population $\rho_{22}(t)$ and (b) $E_2$-level population $\rho_{33}(t)$, where the relaxation time $\gamma$ and temperature are set as $(10 \text{ ps})^{-1}$ and 0 K, respectively. The solid, dashed, and dotted curves represent the cases for coupling strengths, $\hbar U = 100 \mu\text{eV}$, 60, and 40 $\mu\text{eV}$, respectively.

is of the same order as the exciton-phonon coupling in order to obtain the fastest energy transfer in the system.

Figures 12(a) and (b) show the temperature dependence of populations $\rho_{22}(t)$ and $\rho_{33}(t)$, respectively. The temperature is set as either $T = 50$ or 100 K. These results are obtained by using a Laplace transform of Eq. (45), where singular points are derived numerically for the given numerical parameters. As mentioned in the beginning in Sec. 3, a finite temperature induces the incoherent back transfer of energy, and this results in residual populations.
at the upper energy levels in both quantum cubes. Figure 12(a) shows that the $E_2$-level population in QD-B converges on a finite temperature-dependent value that can be derived numerically and is denoted by the horizontal dotted lines in Fig. 12(a). As shown in Fig. 12(b), however, the time evolution of the $E_1$-level population is almost independent of the temperature, except for a decrease in the amplitude because of the residual populations at upper energy levels.

**System tolerance and further discussion**

So far, the theoretical modelling of the population dynamics in a two-quantum-dot system has assumed an ideally perfect resonance condition, which may be too tight to fabricate such a system with definite size ratios. In order to release the resonance condition, we estimate allowable tolerance, or size deviation of quantum dots from designed values. When the deviation from the resonant energy $\hbar \Delta \Omega$ in QD-B is introduced, the factor on the right hand side in Eq. (46b) is modified, and the ratio of that in off-resonance to on-resonance is approximately proportional to $\gamma^2/(\gamma^2 + \Delta \Omega^2)$ ($\gamma/2 \gg U$). Therefore, we can achieve efficiency more than 50% even if the deviation $\Delta \Omega < \gamma$ is introduced. When the dot size and relaxation constant are set as 7.1 nm ($= 5\sqrt{2}$ nm) and $\hbar \gamma = 3$ meV, respectively, approximately 10%-deviation of the dot size can be allowed. As the size of quantum dots is larger, the tolerable deviation is more relaxed. It is feasible to make such quantum dots by the recent advancement of nano-fabrication techniques [28,29]. In fact, experimental results in our research group [30] show the consistent population dynamics as we discussed above.
Our results and discussion might be valid within the Born-Markov approximation. Even if the Born approximation is admitted because of the large volume of the phonon reservoir, the Markov approximation may not be guaranteed at low temperature, as assumed in the transformation in Eq. (43). Therefore, we might need to deal with it more carefully, since it has been pointed out that non-Markov behavior manifests itself at low temperature\[26,31\]. The effects of non-Markov behavior on the dynamics are now under investigation.

We focused our attention on the energy transfer process in the nanometric regime, but it is quite important to investigate signal control, and to manipulate the electronic states of the components in a nanophotonic device, where we must deal with a two-exciton state, as shown in Fig. 6(b). In the following, we show numerical results of exciton population dynamics with signal manipulation in a three-quantum-dot system, which is formulated by the same manner except for phenomenological insertion of control-light pulse.

3.2 Nanophotonic Switch

Switching Operations

In Fig. 13, we show a theoretical model of the nanophotonic switch, in which each neighbouring pair of quantum dots has a resonant energy level that is determined by setting the size as 1(I) : 2(C) : \(\sqrt{2}(O)\), and thus, the system consists of 2-level, 3-level, and 5-level quantum dots depending on the size ratios. In Sec. 3.1, we ignore radiative relaxation in the exciton ground levels because the radiative relaxation time is the order of 1 ns which is enough longer than the non-radiative one. However, for an iterative switching operation discussed here, exciton population has to be swept out to the external field and return into the initial state. This radiative relaxation, i.e. spontaneous emission of photons, is described in the same manner in Eq. (44) based on the Born-Markov approximation for a surrounding photon reservoir, where the radiative relaxation constants are much smaller than the non-radiative ones. In Fig. 13, the notations of radiative and non-radiative relaxation constants are distinguished by the Greek characters \(\gamma\) and \(\Gamma\), respectively, where the subscripts of the relaxation constants represent the terminal quantum dots for the input, output, and control. For simplicity, we assume that photon and phonon reservoirs are both empty \((T = 0)\), where the effect of temperature is just lowering the signal contrast and does not affect the dynamics in the nanophotonic switch. In a switching device, we require another important element, that is injection of external control signal to change the ON and OFF-states. Rigorous modelling of the external exciton excitation is quite difficult since matter coherence presents. Here, we assume weak excitation in which optical nutation or Rabi oscillation due to the control pulse does not appear, and approximately regards the excitation as an incoherent process. Thus, the external excitation can be also written by using the Born-Markov approximation with the very large number of photons. Adding the external
excitation in Eq. (44), a phenomenological equation of motion for the density operator reads

$$\dot{\hat{\rho}}(t) = -\frac{i}{\hbar} [\hat{H}_0 + \hat{H}_{\text{int}}, \hat{\rho}(t)]$$

$$+ (\text{non-radiative relaxation part}) + (\text{radiative relaxation part})$$

$$+ A_{\text{pump}}(t)(\hat{C}_1 \hat{\rho}(t) \hat{C}_1^\dagger + \hat{C}_1^\dagger \hat{\rho}(t) \hat{C}_1),$$

(47)

where $\hat{C}_1$ and $\hat{C}_1^\dagger$ represent the creation and annihilation operators of an exciton at the energy sublevel in QD-C, and the other verbose descriptions for the radiative and non-radiative relaxations are omitted. $A_{\text{pump}}(t) \propto n_{\text{ph}}$ ($n_{\text{ph}}$: photon number) is a time-dependent pumping rate of the control light pulse.

The operation of this switch is as follows: an input signal creates an exciton at the (1,1,1)-level in QD-I, and this transfers to the resonant energy levels of the adjacent two quantum dots via an optical near-field coupling[15]. While some energy levels with even total quantum numbers are optically (dipole) forbidden and are not directly excited by the far-field light, the optical near field allows such transition because their spatial localization resolves the wave function of adjacent nanometric quantum dots far beyond the diffraction limit of light. The ON or OFF-state of the switch corresponds whether the lowest (1,1,1)-level in QD-C is unoccupied or occupied, respectively. When this energy level is occupied (state-filling), the initial excitation in QD-A ultimately reaches the (2,1,1)-level in QD-C or the (1,1,1)-level in QD-O, leading to an output signal. The lowest energy level in each quantum dot is coupled to a free photon reservoir to sweep out the excitation energy radiatively and to a laser photon reservoir to control this device by using a light pulse. Intra-sublevel transitions in QD-C and D guarantee unidirectional energy transfer in the three quantum-dot system.

By using this model, we derived equations of motion for density matrix elements for possible one- and two-exciton states similar to Eq. (45), and calculated the exciton dynamics numerically. As an initial exciton population, a steady state is prepared, where the (1,1,1)-level in QD-I is excited weakly and
Fig. 14. Temporal evolution of the (1,1,1)-level population in QD-O from the initial steady state. The adjustable parameters are set as follows: optical near-field coupling $U = U' = (130 \text{ ps})^{-1}$, radiative relaxation constants $\gamma_i = (16.8 \text{ ns})^{-1}$, $\gamma_c = (2.1 \text{ ns})^{-1}$, and $\gamma_o = (5.9 \text{ ns})^{-1}$, and non-radiative relaxation time $\Gamma_c = (20 \text{ ps})^{-1}$, $\Gamma'_c = (10 \text{ ps})^{-1}$, $\Gamma''_c = (30 \text{ ps})^{-1}$, and $\Gamma_o = (20 \text{ ps})^{-1}$.

continuously. The output signal (luminescence intensity) is proportional to the (1,1,1)-level population in QD-O. Figure 14 shows the temporal evolution of the exciton population in QD-O after an incoherent short pulse (10 ps) is applied to QD-C. This situation corresponds to transition from the OFF-state to the ON-state. When state-filling is achieved instantaneously, the exciton population in QD-O increases and fast vibration appears as shown in Fig. 14. The vibration manifests itself only for the state-filling condition. This is apparently caused by mutation between the (2,1,1) and (1,1,1)-levels in QD-C and QD-O, since the intra-sublevel transition in QD-C is prevented due to the state-filling. In this case, the switching speed from the OFF to ON-states is about 2 ns, which is not so fast. However, as discussed in Sec. 3.1, we expect that the fastest switching time about sub-100 ps can be obtained, when the coupling strength of the optical near field is designed as the same order of non-radiative relaxation constant.

The readers may notice that the relaxation time in the later stage over 2 ns is very slow, which is approximately twice of an isolated quantum dot, and the switch seems to spend much time for recovery to the initial state. This long relaxation time is caused by the coherence between QD-C and QD-O during the state-filling in QD-C. Thus, the phenomenon cannot be reproduced using incoherent rate equation approach, and is a negative property in the nanophotonic switch. One way to realize fast recovery time is that another larger quantum dot which has fast relaxation time is located near QD=C to break the state-filling condition. We also have another interesting idea to use stimulated absorption and emission process for the recovery of the system, which is discussed in the following subsection.
**Operation Using Stimulated Absorption and Emission**

For switching and recovery operations in this system, we have found an important feature related to spontaneous absorption and emission process. At first glance, it seems that the switching device does not return the initial OFF-state unless it spends very long time depending on the radiative lifetime as demonstrated in Fig. 14. However, the exciton population in the system actually decays less than that of the initial steady state during control light injection, and reaches another stable state, where the transition time from the initial state to the second steady state is much faster than the spontaneous emission lifetime as shown in Fig. 15. The effect can be explained by the stimulated absorption and emission in QD-C. For the control light injected, the exciton population in the lower energy sublevel in QD-C approaches by force a half of a unit value due to the stimulated absorption and emission. In the initial steady state, the exciton population in the level is beyond a half value, and thus, the stimulated emission dominantly operates to decrease the population less than the initial steady state, leading to the stable state with lower population. When we stop the control light injection, the exciton population increases beyond that of the beginning steady state with the fast transition time depending on the coupling strength via the optical near field. Using these stimulated absorption and emission process, the fast and iterative switching operations can be achieved.

Figure 16 is one of the calculated results, where two control light pulses with 500 ps-pulse width are applied. We observe that the transition from ON to OFF-states takes only a few 100 ps. With further optimization of the intercube distance, quantum-dot size, and surrounding materials of photon reservoir, we can obtain a switching time (OFF to ON, and ON to OFF) less

![Graph](image_url)

**Fig. 15.** Temporal evolution of the (1,1,1)-level population in QD-O from the initial steady state. The applied control pulse width is set as 2 ns. The same adjustable parameters in Fig. 6 are adopted.
that 100 ps, which is in the order of the inverse of optical near-field coupling strength, and is enough fast for highly integrated nanophotonic devices our proposed.

3.3 Summary

In this section, at first, we have formulated exciton dynamics in a two-quantum-dot system coupled to phonon reservoir, i.e., 2-level and 3-level quantum-dot systems, where energy transfer occurs between energy sublevels with different quantum numbers. Such energy transfer is inactive for the far-field light, and is not realized unless using the optical near-field interaction. Moreover, owing to the fast relaxation via exciton-phonon coupling, unidirectional energy transfer can be realized in a nanometric space, which is difficult in conventional optical devices when miniaturization of the device progresses. With the help of density operator formalism, temporal evolution can be solved analytically for zero temperature. As a result, we have found that an optimal condition exists for the fast switching operation; the coupling strength of an optical near field is comparable to the intra-sublevel relaxation constant. On the other hand, for finite temperature, the excitation from lower sublevels to upper sublevels occurs, and the signal contrast in this switch becomes lower. In order to improve the signal contrast, a mechanism to sweep out the population compulsorily from the output energy level is required. Although we have not mentioned in Sec. 3.1, stimulated absorption and emission process may be one of useful phenomena for an improvement of the signal contrast, which has been described as another viewpoint in Sec. 3.2.
A fundamental switch is generally a three-terminal device for input, output, and control signals. We have investigate a three-quantum-dot system numerically as a nanophotonic switch. The energy transfer dynamics is almost same as a two-quantum-dot system, except for control excitation. When the control light pulse is applied, there are two excitons in the switch, and energy transfer path of an exciton changes due to a state-filling effect. For iterative operation of the nanophotonic switch, a mechanism to sweep out the exciton population is again required. We can use spontaneous emission for this, but the spontaneous lifetime is not so fast as in the order of 1 ns. Here, we have found that the stimulated absorption and emission process are valuable for the fast iterative operations, because the time for sweeping out the exciton population is free from the spontaneous emission lifetime and is determined by the coupling strength of the optical near field. By using these process and optimizing the coupling strength of the optical near field, we have roughly estimated the switching time of sub-100 ps. Such a nanophotonic switch has been already demonstrated experimentally in our research group by using CuCl quantum cubes embedded in a NaCl matrix\cite{30}, where they searched out appropriate quantum-dot trio with the ratios of $1: \sqrt{2}: 2$ by means of a spectroscopic method.

With further progress in nanofabrication techniques, any quantum dots will be aligned at desired positions, and thus, a highly-integrated optical information processing device which consists of the nanophotonic switch can be realized. Furthermore, novel type of devices, which does not operate in a conventional way, should be proposed in the next stage. Our imagined form of such nanophotonic functional devices are composed by a quantum mechanical information processing part as well as a classical dissipative information processing part. In the next section, such type of nanophotonic devices, in which coherently coupled states due to several quantum dots are positively utilized, will be explained analytically. We expect that the reader will feel a large possibility in nanophotonics and future nanophotonic device technologies.

4 Nanophotonic Functional Devices Using Coherently-Coupled States

As we mentioned in Sec. 1, characteristic coupled states via an optical near-field interaction can be generated in a system without dissipation, such as symmetric and anti-symmetric states. These states extend two or more quantum systems with matter coherence as explained in Sec. 1.2, and have different energies in relation with the coupling strength of the optical near field. When we prepare another quantum system which interacts with these states, the energy difference affects resonant conditions of energy transfer via the optical near field. Therefore, the coherently-coupled states can be utilized to pick up some information signal selectively.
In this section, we propose some nanophotonic functional devices, which consist of several quantum dots coupled via an optical near field. Such devices have two key operation parts: one is a coherent operation part or a quantum mechanical operation part, and the other is a dissipative output part (See Fig. 4 in Sec. 1). As we mentioned in the previous sections, unidirectional energy transfer is indispensable in any functional devices for identification of a final state or an output signal, which can be guaranteed by using intra-sublevel relaxation due to exciton-phonon coupling. The dissipative output part includes such unidirectional irreversibility with the help of the energy sublevels in quantum dots. On the other hand, in the coherent operation part, an exciton excitation exists at the resonant energy levels in several quantum dots mediating the optical near field, where the coherently-coupled states survives for a short period of time, before the excitation decays in the dissipative output part[32]. This section focuses on taking full advantage of these coherent operation part and dissipative output parts to realize functional operations based on nanophotonic inherent features[33]. In the former part of this section, as a typical example, we consider a three-quantum-dot system illustrated in Fig. 17. In this system, two identical quantum dots (QD-A and B) are resonantly coupled with each other via an optical near field, that consists of the coherent operation part, and a third quantum dot with larger size than the other two corresponds to the dissipative output part.

Various authors have investigated the coupling properties and dynamics in a pair of quantum dots. For example, the energy shift due to exciton-exciton or Coulomb interactions between electrons and holes has been evaluated the-
oretically to process quantum information, and a controlled-NOT logic gate has been proposed using the energy shift. In these studies, excitons or qubits were controlled by two-color laser pulses of the far-field light. As a similar subject to this section, Quiroga and Johnson theoretically discussed the dynamics in two and three-quantum-dot systems and presented a way to prepare both quantum Bell and Greenberger-Horne-Zeilinger entangled states, by using the far-field light, which allows only global excitation of two and three quantum dots with spatially symmetric arrangement. By contrast, we deal with coupled quantum-dot systems arranged symmetrically and asymmetrically, which are individually excited by the optical near field, and the intra-sublevel relaxation is also considered for the unidirectional energy transfer. Note that the excitation in each quantum dot can be prepared individually owing to the spatial localization of the optical near field. The exciton dynamics driven by the optical near field has been investigated in the case of a coupled two-quantum-dot system with a relaxation process in Sec. 3.1. The energy transfer between two quantum dots is expressed as a Förster-like process, and the nutation of excitation occurs in the strongly coupled or resonant energy levels, corresponding to the coherent operation part in our system. For the short period before relaxation, certain coherently-coupled states appear in the coherent operation part, depending on the initial excitation. In order to prepare the initial excitation, the shorter excitation time in the individual quantum dot than the energy transfer time between two identical quantum dots is necessary, where the excitation time is inversely proportional to the optical near-field intensity. The energy transfer time or coupling strength via an optical near field can be controlled by adjusting interdot distance. The population in the coherently-coupled states can be transferred to the third quantum dot (QD-C) if the energy level of QD-C is adjusted to couple resonantly with the entangled states in the coherent operation part. If this happens, QD-C operates as the dissipative output part, which involves an intra-sublevel relaxation process due to the exciton-phonon interaction. In this manner, unidirectional energy or signal transfer is satisfied.

This section is organized as follows. First of all, we formulate the simplest case of three-quantum-dot system by using the density matrix formalism in Sec. 4.1. Here, to choose appropriate bases, which reflect symmetry of excited states, helps us to catch the physical meanings of the selective energy transfer, and also shows that the selective energy transfer can be controlled by adjusting spatial symmetry of quantum-dot arrangement. In Sec. 4.2, we provide concrete AND and XOR-logic operations in symmetrically arranged quantum-dot systems. Moreover, when the number of quantum dots, which form the coherent operation part, increases, the degrees of freedom for selective energy transfer are extended. As an example, we numerically demonstrate controlled-type logic operations in Sec. 4.3, in which three identical quantum dots are utilized in the coherent operation part. Spatial symmetry is a key
parameter in such a nanophotonic device using coherently-coupled quantum dots because we can realize some strange operations mediating via so-called “dark state”\[38]. Section 4.4 devote to propose a nanophotonic buffer memory by using a spatially asymmetric system, which is an extremely interesting device because the device controls dissipation into a far-field photon reservoir. Furthermore, we show a device for identification of quantum entangled state in Sec. 4.5. Note that these logic and functional operations are in the irreversible process, although quantum entangled states are partially mediated to sort out information about initial excitations. This resembles quantum information processing, however, we do not require long coherence time as a quantum computation. In regarding to quantum information processing with dissipation or decoherence, there are several reports that are discussed tolerance and decoherence-free operations[39,40].

4.1 Dynamics in a Coherently Coupled Quantum-Dot System

In order to examine, conditions of selective energy transfer from the coherent operation part to the dissipative output part, we derive analytic form of equations of motion in a simplest three-quantum-dot system, where two quantum dots are coherently coupled and a larger third quantum dot is assigned as the dissipative output part.

Symmetric and Anti-Symmetric States

Before discussion about exciton population dynamics in a nanophotonic functional device using coherently-coupled states, we explain several appropriate bases in a three-quantum-dot system, which reflect spatial symmetry among the three quantum dots. In such a system, we obtain a clear perspective of exciton dynamics by choosing the bases of the coupled states rather than those of isolated quantum-dot states which have been used in Sec. 3. From the symmetry of the system, the following bases are suitable for describing the dynamics of the one-exciton states using the smallest number of density matrix elements[41]:

\[
\begin{align*}
|1\rangle_s &= \frac{1}{\sqrt{2}} (|e\rangle_A |g\rangle_B |g,g\rangle_C + |g\rangle_A |e\rangle_B |g,g\rangle_C), \\
|1\rangle_a &= \frac{1}{\sqrt{2}} (|e\rangle_A |g\rangle_B |g,g\rangle_C - |g\rangle_A |e\rangle_B |g,g\rangle_C), \\
|1\rangle_{ph} &= |g\rangle_A |g\rangle_B |e,g\rangle_C, \\
|1\rangle_{pl} &= |g\rangle_A |g\rangle_B |g,e\rangle_C,
\end{align*}
\]

where \(|i,j\rangle_C \ (i,j = g,e)\) represents the isolated quantum-dot states in QD-C with upper energy level \(i\) and lower energy level \(j\). One-exciton state describes the condition whereby an exciton exists in either one of the three quantum dots. The crystal ground state and exciton state in each quantum dot, which
is written as $|\Phi_\alpha^o\rangle$ and $|\Phi_\alpha^m(1_x)\rangle$ in Sec. 2, are given by simplified form, such as $|g\rangle_A$ and $|e\rangle_A$, respectively. Similarly, a two-exciton state indicates that two excitons stay in the system. The suitable bases for the two-exciton states without occupation of the lower energy level in QD-C are expressed as

$$|\phi_{2s}\rangle = \frac{1}{\sqrt{2}}(|e\rangle_A|g\rangle_B|e,g\rangle_C + |g\rangle_A|e\rangle_B|e,g\rangle_C),$$  
$$|\phi_{2a}\rangle = \frac{1}{\sqrt{2}}(|e\rangle_A|g\rangle_B|e,g\rangle_C - |g\rangle_A|e\rangle_B|e,g\rangle_C),$$  
$$|\phi_{2b}\rangle = |e\rangle_A|e\rangle_B|g, e\rangle,$$

and those with occupation of the lower energy level are expressed as

$$|\phi_{2s}\rangle = \frac{1}{\sqrt{2}}(|e\rangle_A|g\rangle_B|g, e\rangle_C + |g\rangle_A|e\rangle_B|g, e\rangle_C),$$  
$$|\phi_{2a}\rangle = \frac{1}{\sqrt{2}}(|e\rangle_A|g\rangle_B|g, e\rangle_C - |g\rangle_A|e\rangle_B|g, e\rangle_C),$$  
$$|\phi_{2b}\rangle = |g\rangle_A|g\rangle_B|e, e\rangle,$$

where $|n\rangle_s$ and $|n\rangle_a (n = 1, 2)$ represent symmetric and anti-symmetric states in the coherent operation part, respectively, and the numbers 1 and 2 on the left hand sides in Eqs. (48), (49), and (50) denote the one- and two-exciton states, respectively. In the following, first, we use these bases to investigate the exciton dynamics in a symmetrically arranged three-quantum-dot system. Then, we describe how to realize logic operation devices by using the symmetrically arranged quantum dots. Our discussion can expand into some interesting functional devices using spatial asymmetry of the system, which can be caught by the above characteristic bases.

**Master Equation**

By tracing out the photon degrees of freedom in the optical near-field coupling, which is formulated in Sec. 2, a model Hamiltonian for the three-quantum-dot system $\hat{H}$ is given by

$$\hat{H} = \hat{H}_0 + \hat{H}_{\text{int}},$$

where

$$\hat{H}_0 = \hbar \Omega_A \hat{A}^\dagger \hat{A} + \hbar \Omega_B \hat{B}^\dagger \hat{B} + \hbar \sum_{i=1}^2 \Omega_C_i \hat{C}_i^\dagger \hat{C}_i,$$

$$\hat{H}_{\text{int}} = \hbar U_{AB} (\hat{A}^\dagger \hat{B} + \hat{B}^\dagger \hat{A}) + \hbar U_{BC} (\hat{B}^\dagger \hat{C}_2 + \hat{C}_1^\dagger \hat{B}) + \hbar U_{CA} (\hat{C}_2^\dagger \hat{A} + \hat{A}^\dagger \hat{C}_2),$$

where the definitions of the creation and annihilation operators, $(\hat{A}^\dagger, \hat{A})$, $(\hat{B}^\dagger, \hat{B})$, and $(\hat{C}_i^\dagger, \hat{C}_i)$, are shown schematically in Fig. 18. We assume that
these are fermionic operators to give the effect of exciton-exciton interaction phenomenologically in the same energy level, but we neglect the exciton-exciton interaction between sublevels in QD-C. The eigenfrequencies for QD-A and B, and the optical near-field couplings among three quantum dots are denoted as $U_{AB}$, $U_{BC}$, and $U_{CA}$. The equation of motion for the density operator of the quantum-dot system, $\hat{\rho}(t)$, is expressed by using the Born-Markov approximation[26] as

$$\dot{\hat{\rho}}(t) = -\frac{i}{\hbar} [\hat{H}_0 + \hat{H}_{\text{int}}, \hat{\rho}(t)] + \Gamma \left\{ 2\hat{C}_1^\dagger \hat{C}_2 \hat{\rho}(t) \hat{C}_1^\dagger \hat{C}_2 - \hat{C}_2^\dagger \hat{C}_1 \hat{C}_1^\dagger \hat{C}_2 \hat{\rho}(t) - \hat{\rho}(t) \hat{C}_2^\dagger \hat{C}_1^\dagger \hat{C}_2 \right\}, \quad (53)$$

where the non-radiative relaxation constant due to exciton-phonon coupling is denoted as $\Gamma$. The radiative relaxation due to exciton-photon coupling is omitted because the time scale of the optical near-field coupling and the exciton-phonon coupling is much faster than the radiative lifetime, which is in the order of a few nanoseconds. Taking matrix elements of Eq. (53) in terms of Eq. (48) after substituting Eqs. (51) and (52) into Eq. (53), we obtain the following simultaneous differential equations for the one-exciton states:

$$\dot{\rho}_{s_1,s_1}(t) = \sqrt{2} \hat{U}'(\rho_{s_1,ph_1}(t) - \rho_{ph_1,s_1}(t)) + i(\Delta \Omega - U) \rho_{s_1,s_1}(t) \quad \text{and} \quad \rho_{ph_1}(t) \left( \begin{array}{c} i(\Delta \Omega - U) - \frac{\Gamma}{2} \rho_{s_1,s_1}(t) \\ i\sqrt{2} \hat{U}'(\rho_{s_1,s_1}(t) - \rho_{ph_1,ph_1}(t)) \\ -i\sqrt{2} \hat{U}'\rho_{S_1,A_1}(t) - i\Delta \Omega_{AB}\rho_{A_1,P_1}(t) \end{array} \right), \quad (54b)$$
described in Eq. (49), we obtain
\[ \rho_{\phi_{1, s_1}}(t) = \left\{ -i(\Delta \Omega - U) - \frac{\Gamma}{2} \right\} \rho_{\phi_{1, s_1}}(t) \]
- \sqrt{2} \hat{U}'(\rho_{\phi_{1, s_1}}(t) - \rho_{\phi_{1, \phi_{1}}}(t))
+ i \sqrt{2} \hat{U}'(\rho_{\alpha_{1, s_1}}(t) + i \Delta \Omega_{AB} \rho_{\alpha_{1, s_1}}(t),

\hat{\rho}_{\phi_{1, \phi_{1}}}(t) = -\Gamma \rho_{\phi_{1, \phi_{1}}}(t) - i \sqrt{2} \hat{U}'(\rho_{\phi_{1, s_1}}(t) - \rho_{\phi_{1, \phi_{1}}}(t))
- i \Delta \Omega_{AB} \rho_{\phi_{1, \phi_{1}}}(t),

\hat{\rho}_{\alpha_{1, \phi_{1}}}(t) = \left\{ -i(\Delta \Omega + U) - \frac{\Gamma}{2} \right\} \rho_{\alpha_{1, \phi_{1}}}(t)
- \sqrt{2} \hat{U}'(\rho_{\alpha_{1, \phi_{1}}}(t) - \rho_{\phi_{2, \phi_{1}}}(t))
+ i \sqrt{2} \hat{U}'(\rho_{\alpha_{1, s_1}}(t) - i \Delta \Omega_{AB} \rho_{\alpha_{1, s_1}}(t),

\hat{\rho}_{\phi_{1, \alpha_{1}}}(t) = \left\{ -i(\Delta \Omega + U) - \frac{\Gamma}{2} \right\} \rho_{\phi_{1, \alpha_{1}}}(t)
+ i \sqrt{2} \hat{U}'(\rho_{\phi_{1, \alpha_{1}}}(t) - \rho_{\phi_{2, \alpha_{1}}}(t))
+ i \sqrt{2} \hat{U}'(\rho_{\phi_{1, s_1}}(t) - i \Delta \Omega_{AB} \rho_{\phi_{1, s_1}}(t),

\hat{\rho}_{\alpha_{1, \alpha_{1}}}(t) = \left\{ i(\Delta \Omega + U) - \frac{\Gamma}{2} \right\} \rho_{\alpha_{1, \alpha_{1}}}(t)
- i \sqrt{2} \hat{U}'(\rho_{\alpha_{1, s_1}}(t) - \rho_{\alpha_{1, \alpha_{1}}}(t)),

\text{where the density matrix element } \alpha \langle \alpha | \hat{\rho}(t) | \beta \rangle \text{ is abbreviated } \rho_{\alpha_{\alpha_{\beta_{\beta}}}}(t) \text{ and the parameters, which have a dimension of frequency and characterize the dynamics in this system, are defined as } \Delta \Omega = \Omega_{C_{2}} - (\Omega_{A} + \Omega_{B})/2, \Delta \Omega_{AB} = \Omega_{A} - \Omega_{B}, \hat{U} = (U_{BC} + U_{CA})/2, \text{ and } \Delta \hat{U} = (U_{BC} - U_{CA})/2. \text{ The optical near-field coupling between QD-A and B, which is in a coherent operation part, is rewritten as } U = U_{AB}. \text{ Similarly, in the case of two-exciton states described in Eq. (49), we obtain}

\hat{\rho}_{\phi_{2, \phi_{2}}}(t) = -\Gamma \rho_{\phi_{2, \phi_{2}}}(t) + i \sqrt{2} \hat{U}'(\rho_{\phi_{2, \phi_{2}}}(t) - \rho_{\phi_{2, \phi_{2}}}(t))
+ i \Delta \Omega_{AB} \rho_{\phi_{2, \phi_{2}}}(t) - \rho_{\phi_{2, \phi_{2}}}(t),

\hat{\rho}_{\phi_{2, \phi_{2}}}(t) = \left\{ -i(\Delta \Omega + U) - \frac{\Gamma}{2} \right\} \rho_{\phi_{2, \phi_{2}}}(t)
+ i \sqrt{2} \hat{U}'(\rho_{\phi_{2, \phi_{2}}}(t) - \rho_{\phi_{2, \phi_{2}}}(t))
+ i \sqrt{2} \hat{U}'(\rho_{\phi_{2, \phi_{2}}}(t) - i \Delta \Omega_{AB} \rho_{\phi_{2, \phi_{2}}}(t),

\hat{\rho}_{\phi_{2, \phi_{2}}}(t) = \left\{ i(\Delta \Omega + U) - \frac{\Gamma}{2} \right\} \rho_{\phi_{2, \phi_{2}}}(t)
- i \sqrt{2} \hat{U}'(\rho_{\phi_{2, \phi_{2}}}(t) - \rho_{\phi_{2, \phi_{2}}}(t))
where \( U_{\Delta \Omega} \) is the energy level in QD-C, where the energy shift \( \Delta \Omega \) is determined by the strength of the optical near-field coupling. The energy level \( E_{\text{QD-C}} \) is calculated as

\[
E_{\text{QD-C}} = E_{\text{QD-A}} + U_{\Delta \Omega}.
\]

From Eqs. (54) and (55), we can attract three important features with regard to energy transfer from the coherent operation part to the dissipative output part. Comparing Eqs. (54b) with (54c), where the diagonal elements represent the transition probability, opposite contributions appear in the first terms, i.e., \( \Delta \Omega - U \) and \( \Delta \Omega + U \). These correspond to the difference of resonance conditions for symmetric and anti-symmetric states, respectively. The resonance conditions are easily interpreted by considering the energies for the symmetric and anti-symmetric states, which can be derived from Eqs. (51) and (52) as

\[
\langle s_1 | \tilde{H} | s_1 \rangle = \hbar (\tilde{\Omega}_{AB} + U), \tag{56}
\]
\[
\langle a_1 | \tilde{H} | a_1 \rangle = \hbar (\tilde{\Omega}_{AB} - U), \tag{57}
\]

where \( \tilde{\Omega}_{AB} = (\Omega_A + \Omega_B)/2 \). Figure 19(a) is the schematic illustration that explains relation between isolated quantum-dot states and coherently-coupled states. A filled circle indicates that an exciton occupies the corresponding energy level, while a semicircle indicates that an exciton exists in the energy level in either QD-A or B with a certain probability. The first feature is that we can select the symmetric and anti-symmetric states by adjusting the energy level in QD-C, where the energy shift \( \Delta \Omega \) is determined by the strength of the optical near-field coupling \( U \).
Fig. 19. Schematic explanation of a relation between isolated quantum-dot states and coupled states for (a) one- and (b) two-exciton states. The left and right illustrations represent the initial and final states, respectively, and the center figures denote the coupled states for input and output. The energy transfer between symmetric states $|n⟩_s$ and output states $|n⟩_{ph}$, where $n = 1, 2$, occurs via coupling strength $\hbar U'$, and that between asymmetric states $|n⟩_a$ and output states $|n⟩_{ph}$ does via $\hbar \Delta U'$.

The second feature is also observed from the second terms in Eqs. (54b) and (54e). These terms determine the strength or the speed of resonant energy transfer between the coherent operation part and dissipative output part. The difference is as follows; the symmetric state resonantly couples to the upper energy level in QD-C, i.e., $|1⟩_{ph}$-state, by mediating the averaged coupling strength $U'$, while the anti-symmetric state does by mediating the difference between two coupling paths, $\Delta U'$. When the three quantum dots are arranged symmetrically in a nanometric space, the energy transfer between $|1⟩_a$-state and $|1⟩_{ph}$-state is forbidden because of $\Delta U' = 0$, in other words, $\langle 1 | 1 | \hat{H}_{\text{int}} | 1⟩_a = 0$. Therefore, we can control the energy transfer or signal flow by using spatial symmetry in the system.

The third feature is related to the exciton numbers in the system, which is interpreted by comparing Eqs. (54b) and (55b), or Eqs. (54e) and (55e). Similar to the first feature, the resonance conditions for the energy transfer from the coherent operation part to the dissipative output part indicate opposite contribution, i.e., $\Delta \Omega - U$ and $\Delta \Omega + U$ for one- and two-exciton states, respectively. In Fig 19(b), the correspondence of isolated quantum-dot states and coupled states are schematically drawn, where we can see the inversion
of the resonance conditions. The difference of the resonance conditions is applicable to two input logic operations discussed below. Note that the first two features, which are discussed for only one-exciton state, are reversed in the case of two-exciton state as you observed in Eq. (55).

4.2 Nanophotonic Logic Gates

In order to realize well-known AND- and XOR-logic operations, we assume spatially symmetric quantum-dot system, and thus, the related differential equations in Eqs. (54) and (55) are restricted, which decouples the antisymmetric \( |1\rangle_a \) and \( |2\rangle_a \)-states from the above. In this case, the dynamics can be solved analytically with the help of Laplace transforms for typical initial conditions. The output population for the one-exciton state can be written as

\[
\rho_{ph_1,pl_1}(t) = \Gamma \int_0^t \rho_{ph_1,ph_1}(t')dt' = \frac{1}{2} + \frac{4U'^2}{\omega_+^2 - \omega_-^2} e^{-\Gamma/2} \left( \cos \phi_+ \cos(\omega_+ t + \phi_+) - \cos \phi_- \cos(\omega_- t + \phi_-) \right),
\]

with

\[
\omega_\pm = \frac{1}{\sqrt{2}} \left[ (\Delta \Omega - U)^2 + W_+ W_- \right]^{1/2},
\]

\[
\phi_\pm = \tan^{-1} \left( \frac{2\omega_\pm}{\Gamma} \right),
\]

\[
W_\pm = 2\sqrt{2} U' \pm \Gamma / 2.
\]

for the initial condition \( \rho_{s_1,s_1}(0) = \rho_{a_1,a_1}(0) = \rho_{s_1,a_2}(0) = \rho_{a_1,s_1}(0) = 1/2 \), which corresponds to the condition \( A \langle e | B \langle g | C \langle g | g | \hat{\rho}(t) | e \rangle A | g \rangle B \langle g, g \rangle C = 1 \) and otherwise zero. The notation of optical near-field coupling is rewritten as \( U' = \overline{U}' \) because the coupling strengths between QD-B and C, and QD-C and A are equivalent for symmetrically arranged system, that is \( U' = U_{BC} = U_{CA} \). The first line in Eq. (58) denotes the irreversible process of non-radiative relaxation, which is easily interpreted from the temporal sequence of the one-exciton state illustrated in Fig. 20(a).

Analytic solutions for two-exciton states can be obtained from an equation similar to Eq. (58), except for the sign of \( U \), i.e., with the resonance conditions inverted. The probability of an exciton occupying the lower energy level in QD-C is
Fig. 20. Schematic illustration of temporal sequences at the final output stages in the cases of (a) one-exciton state and (b) Two-exciton state.

\[
\rho_{s,sl}(t) + \rho_{pl,pl}(t) \Gamma \int_0^t \rho_{sh,sh}(t')dt' = 2 \left[ \frac{1}{2} + \frac{4U'^2}{\omega'^2 - \omega'^2 - \Gamma/2} e^{-(\Gamma/2)t} \right. \\
\left. \times \{ \cos \phi'_+ \cos(\omega'_+ t + \phi'_+) - \cos \phi'_- \cos(\omega'_- t + \phi'_-) \} \right], \quad (60)
\]

with

\[
\omega'_\pm = \frac{1}{\sqrt{2}} \left[ (\Delta \Omega + U)^2 + W_+ W_- \right. \\
\left. \pm \sqrt{\{(\Delta \Omega + U)^2 + W_+^2\} \{ (\Delta \Omega + U)^2 + W_-^2 \}} \right]^{1/2},
\]

\[
\phi'_\pm = \tan^{-1} \left( \frac{2\omega'_\pm}{\Gamma} \right), \quad (61)
\]

where the factor 2 in Eq. (60) comes from the initial conditions for the two-exciton state, i.e., \( \rho_{ph,ph}(0) = 1 \) and otherwise zero. As you can see from the right hand side in the first line in Eq. (60), there are two final output states, \( \ket{2}_{sl} \) and \( \ket{2}_{pl} \). However, the previous excited state, \( \ket{2}_{sh} \), only contributes to the output population, because the state \( \ket{2}_{pl} \) is made from \( \ket{2}_{sl} \), and thus, the total population does not change, which is illustrated in Fig. 20(b). In the both of Eqs. (58) and (60), the second terms with the denominators \( \omega'^2 - \omega'_- \) and \( \omega'^2 - \omega'_- \), respectively, contribute to the increase of population in the output energy levels. When we set \( \Delta \Omega = U \), efficiency of energy transfer is dominant for the one-exciton state because \( \omega'^2 - \omega'^2 \) becomes minimum, while that is dominant for the two-exciton state in the case of \( \Delta \Omega = -U \). These are analytical description of resonant energy transfer depending on the number.
of input excitons and a basic idea for logic operation. In the following, we explain AND- and XOR-logic operations schematically, and show concrete calculated results of them.

**AND-Logic Operation**

When the upper energy level in QD-C is negatively shifted, which corresponds to $\hbar(\Omega - U)$, we can realize an AND-logic gate. Figure 21 represents energy diagram in the system with negative detuning in QD-C. As you can see from Fig. 21(a), the resonant energy transfer occurs only for the two-exciton state via the symmetric state in QD-A and QD-B, while the energy transfer for the one-exciton state does not because the energy level in QD-C is resonant to the anti-symmetric state which is dipole inactive state for the symmetrically arranged quantum-dot system. This characteristic selective energy transfer assures an AND-logic operation.

The temporal evolution of the exciton population on the lower energy level in QD-C, which is analytically derived in Eqs. (58) and (60), is plotted in Fig. 22, where the strengths of optical near-field coupling, $\hbar U = 89 \mu$eV and $\hbar U' = 14 \mu$eV, referred to the estimated values in Sec. 2 for CuCl quantum cubes embedded in NaCl matrix. As we discussed above, the coupling to far-field light is neglected because our interests are fast population dynamics due to the optical near-field coupling, which is in the order of sub-100 ps. In Fig. 22, the exciton population is almost occupied at about 100 ps for the two-exciton state, which is determined by the coupling strength $\hbar U'$ between QD-A and C (QD-B and C). The negative energy shift is also set as 89 $\mu$eV, and the non-radiative relaxation is 10 ps.

From Fig. 22, the output population can be observed only for the two-exciton state as we expected, because the coupling occurs via the symmetric state. On the other hand, in the case of the one-exciton state, the population increases very slowly. This is caused by the weak coupling between the symmetric state in the input side and the output state. The state-filling
time is much longer than spontaneous emission lifetime, and thus, the exciton population for the one-exciton state does not affect the output signal, that is OFF-level. In this manner, these operations for the one- and two-exciton states surely correspond to the AND-logic gate whose size is much smaller than the diffraction limit of light.

**XOR-Logic Operation**

Opposite to the AND-logic gate, suppose a system in which the upper energy level in QD-C is positively shifted, i.e., $\hbar(\Omega + U)$. Energy diagram in this system is illustrated in Fig. 23. In this case, the symmetric and anti-symmetric states for one and two excitons satisfy the conditions for an XOR-logic gate. The energy transfer from the input system to the output system allows when an exciton is excited in either QD-A or QD-B, while, for the two-exciton state, the anti-symmetric state in the output system is dipole inactive against the input state in the symmetrically arranged quantum-dot system.

Figure 24 shows analytic curves of the temporal evolution for the one- and two-exciton states. The given parameters, such as the coupling strength via the optical near field, non-radiative relaxation time, are completely the same as those in Fig. 22. In Fig. 24, the output population appears for the one-exciton state, and the state-filling time is determined by the coupling strength $U'$ between QD-A and C (QD-B and C), which is the same as the AND-logic gate. The readers immediately put the question why the exciton population reaches a half of the maximum for the one-exciton state in the XOR-logic gate. This is because a one-side quantum dot is locally excited as an initial condition, in which both the symmetric and anti-symmetric states
are simultaneously excited with the same occupation probability as described in Eqs. (48a) and (48b). On the other hand, we can observe slow increase of exciton population for the two-exciton state (OFF-level) in Fig. 22, which is is as twice fast as that for the AND-logic gate. This also originates from the initial excited state; the symmetric state in the input system is occupied a half of the maximum at the initial time because of local excitation, while a full of the initial population for the two-exciton state can couple to QD-C in the case of positive energy shift.

The above behavior that the output population is detected when a one-side quantum dot in the input system is initially excited corresponds to an XOR-logic gate as a stochastic meaning. This means that the fully occupied output state cannot achieve for a single exciton process.
Here, we summarize the operation of our proposed AND- and XOR-logic gates in Table 1, which are inherent operations in nanophotonic devices using typical coherent and dissipative process. The system behaves as an AND-logic gate when the energy difference is set to $\Delta \Omega = -U$, and the system provides an XOR-like-logic operation when $\Delta \Omega = U$. It is noteworthy that these operations are different from the quantum logic operation, because long quantum coherence time is unnecessary. The critical limit of these logic gates is determined by the following condition; the energy transfer time from the coherent operation part to the dissipative output part, which is estimated about 50 ps for the CuCl quantum-cubes system, is enough shorter than the radiative lifetime ($\sim 1$ ns) of excitons in each quantum dot.

**Signal Contrast**

The steepness of the resonance determines the contrast of the output signal. In order to discuss how to obtain high contrast signal, dependence of the exciton population on the energy shift $\Delta \Omega$ in the above symmetrically arranged quantum-dot system is plotted in Fig. 25. The longitudinal axis is the population at $t = 100$ ps, which is the time until energy transfer almost finishes under resonance conditions $\Delta \Omega = \pm U$ and is analytically derived from Eqs. (58) and (60). We clearly find that two types of switching operations, i.e., AND- and XOR-logic operations, can be realized by choosing the energy shift as $\Delta \Omega = \pm U$. Here, the peak width of both curves in Fig. 25 is given by the product $W_+ W_-$ in Eqs. (59) or (61), which corresponds to a balance between the coupling strength $U'$ between the coherent operation part and the dissipative output part and the non-radiative relaxation constant $\Gamma$. Therefore, narrow peaks are obtained when the conditions $W_+ \ll 1$ and $W_- = 0$, i.e., $2\sqrt{2}U' \sim \Gamma/2 \ll 1$, are satisfied. In this case, the highest contrast of the logic operations can be achieved.

**Effects of Asymmetry**

It is valuable to examine the exciton dynamics in an asymmetrically ar-
Fig. 25. Variation in the output populations at a fixed time of $t = 100$ ps as a function of the energy shift $\Delta \Omega$. The solid and dashed curves represent the one- and two-exciton states, respectively. The optical near-field coupling strengths $\hbar U$ and $\hbar U'$, and the non-radiative relaxation constant $\Gamma$ have the same values as in Fig. 22.

ranged quantum-dot system to estimate the fabrication tolerance for the system described above and to propose further functional operations inherent in nanophotonic devices which we will discuss in Sec. 4.4 and 4.5. Here, we investigate the effects of asymmetry numerically. In addition, we comment on a positive use of these effects. When the three quantum dots are arranged asymmetrically, we must consider the dynamics of all density matrix elements given in Eqs. (54) and (55), because the exciton population leaks to the antisymmetric states, which decouples in the case of symmetrically arranged system. In the asymmetrically arranged system, the exciton dynamics between states $|n\rangle_s$ and $|n\rangle_{ph}$ do not change from the symmetrically arranged system, where the coupling strength is replaced by the average value $\bar{U}'$. The main difference is that the matrix elements for states $|n\rangle_a$ can couple with states $|n\rangle_s$ and $|n\rangle_{ph}$ in the asymmetrically arranged system, while these are decoupled in the symmetrically arranged system. Two types of coupling emerge in the asymmetric system: one originates from the energy difference $\Delta \Omega_{AB}$ between QD-A and B, and the other comes from the arrangement of the three quantum dots, which is expressed using the parameter $\Delta U'$. Previously\cite{7}, we discussed the influence of the energy difference on the exciton dynamics in a two-quantum-dot system that mainly degrades the signal contrast. Here, we focus on the effects of asymmetry due to the spatial arrangement of each quantum dot, by assuming $\Delta \Omega_{AB} = 0$.

In order to examine the effects of the quantum-dot arrangement, the average coupling strength $\bar{U}'$ is fixed so that states $|n\rangle_s$ and $|n\rangle_{ph}$ maintain the same temporal evolution that was found in the symmetrically arranged system. Then, the difference between the coupling strengths $\Delta U'$ varies from 0 to $\pm U$, where the exciton dynamics are independent of the sign of $\Delta U'$. 
Fig. 26. Temporal evolution of the output populations where the energy difference is set to $\Delta \Omega = -U$ ($\hbar U = 89 \mu eV$). Parts (a) and (b) show the populations for one- and two-exciton states, respectively. The solid, dashed, and dotted curves represent the results for asymmetry factors $\Delta U'/\bar{U}' = 0, 0.5, \text{ and } 1.0$, respectively, where the average coupling strength is set to $\hbar \bar{U}' = 14 \mu eV$. In part (b), the three curves are almost identical.

Therefore, an asymmetry factor is defined by the ratio of $|\Delta U'|$ to $\bar{U}'$, varying from 0 (symmetry) to 1 (maximum amount of asymmetry).

Figure 26 presents the temporal evolution of the output population for the energy shift $\Delta \Omega = -U$ (an AND-logic gate case) with and without an asymmetric arrangement. For the one-exciton state (Fig. 26(a)), the asymmetric arrangement strongly affects the exciton dynamics, and the OFF-state in the AND-logic gate operation is no longer valid because the off-resonance condition for the energy transfer between states $|1\rangle_s$ and $|1\rangle_{ph}$ acts oppositely to the resonance condition between states $|1\rangle_a$ and $|1\rangle_{ph}$. This is evident in Eq. (54), for example, by comparing the matrix elements $\rho_{s1,ph1}(t)$ with $\rho_{a1,ph1}(t)$. Therefore, in the one-exciton state, the exciton population is very sensitive to the asymmetric arrangement. By contrast, the two-exciton state is not influenced by the quantum-dot arrangement (see Fig. 26(b)). We also observe small and high-frequency oscillations for the dashed and dotted curves ($|\Delta U'|/\bar{U}' = 0.5$ and 1.0) in Fig. 26(a). These come from the coherence between states $|1\rangle_s$ and $|1\rangle_a$ which can be seen in the equations of motion of $\rho_{s1,a1}(t)$ and $\rho_{a1,s1}(t)$. Since the coherence is always excited by mediating state $|1\rangle_{ph}$, and the state $|1\rangle_{ph}$ has a short lifetime dominated by the relaxation constant $\Gamma$, the oscillations have no relation to the population dynamics. Figure 27 shows the variation in the output population at $t = 100$ ps as a function of the asymmetry factor $|\Delta U'|/\bar{U}'$. From this figure, it follows that the asymmetry only affects the one-exciton state, where it breaks the OFF-state in the logic gate, as shown by the curve with squares, and the signal contrast decreases with increasing asymmetry.

Conversely, for the XOR-logic gate ($\Delta \Omega = U$), the two-exciton states correspond to the off-resonant states in the symmetric system. Therefore, the
excitation is transferred to the output energy level in QD-C as the asymmetry factor increases, as shown in Fig. 28(b). Similarly, the variation in the output population with the asymmetry factor is plotted in Fig. 29, where the time is fixed at $t = 100$ ps. The figure shows that the XOR-logic operation in the symmetric system is reversed when the asymmetry factor exceeds 0.5 because a one-exciton state can occupy the initial state of $|1\rangle_s$ with a probability of 1/2, as shown above. Consequently, the output population also reaches a probability of 1/2. This is also valid in the asymmetric system. However, the asymmetric arrangement enables coupling of the two-exciton states, $|2\rangle_{ph}$ and $|1\rangle_{ah}$. State $|2\rangle_{ph}$ can be fully excited in the initial stage, so the output population reaches a unit value via states $|2\rangle_{ah}$. This exceeds the output population 0.5 for a one-exciton state with a larger amount of asymmetry.

Although spatial asymmetry acts negatively for the above logic operations by using a symmetrically arranged system, while this has hidden potential toward nanophotonic inherent functions. As mentioned above, the effect of asymmetry is based on coupling to states $|n\rangle_a$ in an asymmetrically arranged system, which are so-called “dark states” [38]. If we create such dark states by using optical near-field interaction just as we intended, which cannot be excited by the far-field light, confinement of photons in a nanophotonic device can be realized, which is discussed in the following. Furthermore, in such an asymmetrically arranged system with via coherent excitations, the symmetric and anti-symmetric states can be excited partially, where both states are expressed by the superposition of eigenstates in isolated (non-interacting) quantum dots. Therefore, a system composed of three quantum dots can not only select information that depends on the initially prepared excitations, but also information that reflects the initial quantum entangled states in the
Fig. 28. Temporal evolution of the output populations for an the energy difference of $\Delta \Omega = U$ ($\hbar U = 89 \mu eV$). Parts (a) and (b) show the populations for the one- and two-exciton states, respectively. The solid, dashed, and dotted curves represent the results for asymmetry factors $\Delta U'/U' = 0, 0.5$, and 1.0, respectively, where the average coupling strength is set to $\hbar \bar{U}' = 14 \mu eV$.

Fig. 29. Variation in the output populations at a the fixed time of $t = 100$ ps as a function of the asymmetry factor, where the energy difference is set to $\Delta \Omega = U$ ($\hbar U = 89 \mu eV$) and an average coupling strength of $\hbar \bar{U}' = 14 \mu eV$ is used. The curves shown with square and circular dots represent the one- and two-exciton states, respectively. The exciton population in the two-exciton state exceeds that of the one-exciton state when the asymmetry factor $\Delta U'/U'$ exceeds 0.5, so the XOR-logic operation is reversed.

coherent operation part. From this perspective, such nanophotonic devices are useful in connecting quantum devices as a detector and interface devices which identify occupation probability of the quantum entangled states in an input signal.

4.3 Nanophotonic Controlled Logic Gates

Up to this point, we have instructively discussed the simplest system with three quantum dots. Focused on a coherent operation part, the readers eas-
ily understand that more coupled states can be prepared when the number of quantum dots increases; for example, the coherent operation part which consists of three identical quantum dots, i.e., a four-quantum-dot system illustrated in Fig. 30(a), has three coupled states,

\[ |1\rangle_{su} = \frac{1}{\sqrt{2}}(|e\rangle_A|g\rangle_B|g\rangle_C + \sqrt{2}|g\rangle_A|e\rangle_B|g\rangle_C + |g\rangle_A|g\rangle_B|e\rangle_C), \]

\[ |1\rangle_a = \frac{1}{\sqrt{2}}(|e\rangle_A|g\rangle_B|g\rangle_C - |g\rangle_A|g\rangle_B|e\rangle_C), \]

\[ |1\rangle_{sl} = \frac{1}{2}(|e\rangle_A|g\rangle_B|g\rangle_C - \sqrt{2}|g\rangle_A|e\rangle_B|g\rangle_C + |g\rangle_A|g\rangle_B|e\rangle_C), \]

for the one-exciton states, and

\[ |2\rangle_{su} = \frac{1}{2}(|e\rangle_A|e\rangle_B|g\rangle_C + \sqrt{2}|e\rangle_A|g\rangle_B|e\rangle_C + |g\rangle_A|e\rangle_B|g\rangle_C), \]

\[ |2\rangle_a = \frac{1}{2}(|e\rangle_A|g\rangle_B|g\rangle_C - |g\rangle_A|g\rangle_B|e\rangle_C), \]

\[ |2\rangle_{sl} = \frac{1}{2}(|e\rangle_A|e\rangle_B|g\rangle_C - \sqrt{2}|e\rangle_A|g\rangle_B|e\rangle_C + |g\rangle_A|e\rangle_B|g\rangle_C), \]

for the two-exciton states. A three-exciton state is degenerated since the energy levels in all three quantum dots are occupied. By using these coupled states, we can propose a characteristic functional device, which is named controlled AND- and XOR-logic gates. Although equations of motion for density matrix elements build up in a similar manner to Eqs. (54) and (55), it is tedious to describe differential equations for all density matrix elements. Here we explain energy transfer property by using schematic illustration. The energy diagram is illustrated in Fig. 30(b), where the energies for one- and two-exciton states are divided into three levels with eigenenergies, \( \hbar \Omega + \sqrt{2}, \hbar \Omega, \) and \( \hbar \Omega - \sqrt{2} \) due to the optical near-field coupling. Note that the coupled state with middle energy is dipole inactive state in a symmetric arranged system [Fig. 30(a)]. The left side in the diagram corresponds to the input (initial) state, where only the three-exciton state is degenerated, while the right denotes the output state in which the three-exciton state becomes non-degenerated since an exciton stays in QD-D without going back to the coherent operation part because of fast intra-sublevel relaxation.

In Fig. 31, we show the result of output population at 300 ps as a function of the energy shift \( \Delta \Omega \), where initial states are set as \( (1,0,0) \), \( (1,1,0) \), and \( (1,1,1) \), the notation corresponding to the quantum-dot label of \( (A,B,C) \). In order to obtain clear energy splitting (sharp resonance) for the one-, two-, and three-exciton states, the strength of optical near-field coupling is set as a suitably optimized value of \( \hbar U = 66 \mu \text{eV} \) (10 ps). This seems somewhat strong as compared with the previously estimated value for CuCl quantum cubes embedded in a NaCl matrix. Therefore, we require some optimization in materials and size of quantum dots to realize this controlled-logic devices.
Fig. 30. (a) Schematic illustration of four-quantum-dot system, which consists of three identical quantum dots as the coherent operation part and a quantum dot with energy sublevels as the dissipative output part. (b) Energy diagram in the four-quantum-dots system. The left and right diagrams correspond to the input and output states, respectively, and the energy are splitted into three levels with the energy shift of $\sqrt{2}U$ for the one- and two-exciton states in the input state and for the two- and three-exciton states in the output state.

Fig. 31. Variation in the output populations at a fixed time of $t = 300$ ps as a function of the energy difference $\Delta \Omega$. The solid, dashed, and dotted curves represent the one-, two-, and three-exciton states, respectively. The optical near-field coupling strengths $\hbar U$ and $\hbar U'$, and the non-radiative relaxation constant $\Gamma$ are set as $66 \, \mu eV$, $3.3 \, \mu eV$, and $(50 \, ps)^{-1}$, respectively.

From Fig. 31, we understand that only two exciton states can coupled to the output state, when we choose the upper energy level in QD-D equivalent to the middle energy level in the coupled states, no energy shift being applied. Regarding to the input terminals as QD-A and B, the exciton population transfers to the output terminal of QD-D differently, whether the control terminal of QD-C is excited or not. In other words, QD-C plays a role to exchange the AND- and XOR-logic operations.
Fig. 32. Temporal evolution of the output populations for possible initial states, \((A, B, C) = (1, 0, 0), (0, 1, 0), (1, 1, 0)\) (without control signal), and \((1, 0, 1), (0, 1, 1), (1, 1, 1)\) (with control signal). The upper energy level in QD-D is adjusted equal to the energy of the other three quantum dots, and the optical near-field coupling strengths \(\hbar U\) and \(\hbar U'\), and the non-radiative relaxation constant \(\Gamma\) are set as \(U = 66\ \mu\text{eV}, \ hU' = 3.3\ \mu\text{eV},\) and \((50\ \text{ps})^{-1}\), respectively.

In the following, we discussed temporal evolution of exciton population on the output energy level in QD-D, which is numerically derived by using bases of isolated quantum dots. Figure 32 shows the results of calculation for all possible initial excitations in the symmetrically arranged four-quantum-dot system. The parameters are set as the same values in Fig. 31. At first, we focus on the exciton dynamics when the population in QD-C (control dot) is empty at the initial time. In this case, an exciton which is prepared in QD-A or QD-B cannot move to the output energy level in QD-D. The reason is as follows; a locally excited state in QD-A is expressed as a superposition of the coupled states, \(|e\rangle_A |g\rangle_B |g\rangle_C = (|1\rangle_{su} + \sqrt{2} |1\rangle_a + |1\rangle_{sl})/2\), where the output energy level is resonant for the state \(|1\rangle_a\) and off-resonant for the states \(|1\rangle_{su}\) and \(|1\rangle_{sl}\) as you can see in Fig. 30. However, the state \(|1\rangle_a\) is dipole inactive for the symmetrically arranged system, and thus, the energy transfer does not occur for the one-exciton state as we can observe the curves labelled as \((1,0,0)\) and \((0,1,0)\) in Fig. 30. On the other hand, when both of QD-A and QD-B are initially excited, \(i.e., (1,1,0)\), the output signal appears because three pair of energy levels in the input and output states completely resonant. Here we emphasize that the states \(|2\rangle_a |g\rangle_D\) and \(|1\rangle_a |e\rangle_D\) can also couple with each other because they have same symmetries.

Second, we pay attention to the case that an exciton is initially prepared in QD-C. In this case, the output population only appears when either QD-A or QD-B is initially excited, which shows similar dynamics to the initial condition of \((1,1,0)\). The readers notice that the curves for the initial \((0,1,1)\) and \((1,0,1)\)-states are slightly different. This is caused by the symmetry of the \((1,0,1)\)-state, while the \((1,1,0)\)-state is asymmetric. When both of QD-A and QD-B are excited, that is the three-exciton state labelled as \((1,1,1)\), the
Table 2. Relationship between the input and output populations for the four-quantum-dot system.

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signal becomes enough small, which is the OFF-level in this logic gate. The above operations are summarized in Table 2. We clearly understand that the four-quantum-dot system operates as an AND- and XOR-logic gates in the cases without and with the control signal, respectively.

Such a nanophotonic device is quite interesting because two types of operations are carried out in a same quantum-dot system, which has large advantages to avoid complex nanofabrication process, and lowering the number of device elements in an integrated nanophotonic circuit.

4.4 Nanophotonic Buffer Memory

As mentioned in Sec. 4.3, we discuss realization of characteristic nanophotonic devices in the remaining sections, in which the coupling features of anti-symmetric states. Since the anti-symmetric states have no total dipole, it cannot be excited by he far-field light, and also cannot radiate the far-field light. Using this dipole inactive feature, we can realize an interesting operation, in which exciton-polariton or incident photon energy is retained in the system for a long time. We refer to this type of device as photon storage or photon buffer memory.

The three-quantum-dot system is the simplest configuration to obtain a completely anti-symmetric state for the photon buffer memory, which is illustrated in Fig. 33. The upper energy level in QD-C is positively shifted by $\Delta \Omega = U$, which is the resonance condition between the input and output parts in the asymmetrically arranged quantum-dot system. For this system, the anti-symmetric state for the one-exciton state is not directly excited, because both of the symmetric and anti-symmetric states are simultaneously
excited. However, if the two-exciton state is initially excited in the input part, we can create completely anti-symmetric state. This is explained by using temporal sequence of the energy states as illustrated in Fig. 34. According to Eq. (55), the energy transfer from the two-exciton states $|2\rangle_{ph}$ to $|2\rangle_{Ah}$ is active when QD-C is asymmetrically located and the upper energy level in QD-C is positively shifted by $\Delta \Omega = U$. An exciton transfers into the upper energy level in QD-C, resonantly, where the rest exciton in the coherent operation part completely stays in the anti-symmetric state. Then, the exciton on the upper energy level in QD-C drops into the lower energy level via intra-sublevel relaxation with leaving the anti-symmetric state in QD-A and QD-B. The lower energy level in QD-C is dipole active state for the far-field light, therefore, the exciton annihilates due to spontaneous emission, which spends several ns, while the anti-symmetric state does not couple to the far-field light because the total dipole moment in a coherent operation part is zero. Moreover, the anti-symmetric one-exciton state is off-resonant to the upper energy level in QD-C, and thus, very long lifetime is expected. In this manner, a complete anti-symmetric state for one-exciton state can be obtained in such a maximally asymmetric system.

Figure 35 represents a numerical result of the time evolution derived by using Eqs. (54) and (55). The solid curve in Fig. 35 corresponds to the sum of exciton population for the states $|1\rangle_a$, $|2\rangle_{ah}$, and $|2\rangle_{al}$, which holds zero total dipole moment for the far-field light. On the other hand, the sum of exciton population in the states, $|1\rangle_s$, $|2\rangle_{sh}$, and $|2\rangle_{sl}$ is plotted by the dashed curve in Fig. 35, which is almost zero value since the initial stage. Actually, the far-field radiation cannot be restrained completely in this system because the weak coupling from the anti-symmetric state to the symmetric state exists, where the energy transfer depends on the average coupling strength $U'$ and the difference of the couplings $\Delta U'$ as shown in Eq. (54). The oscillating behavior is observed for the both curves in Fig. 35, which is also caused by the coupling between the anti-symmetric and symmetric states. Although we have neglected the coupling to the far-field light in this calculation, the slow
Fig. 34. Temporal sequence of energy states in a three-quantum-dot system, which starts from two-exciton state. $\hbar \Omega_{C_1, C_2}$ denotes the energy difference between upper and lower energy levels in QD-C.

exponential decay could be observed in the anti-symmetric state when we take into account the far-field coupling.

The above discussion is devoted to a writing process in the photon buffer memory, or how to prepare an anti-symmetric state in a coupled quantum-dot system. However, in a general system, we need to consider a mechanism for reading information in addition to the writing process, which will be discussed elsewhere.

4.5 Nanophotonic Signal Splitter for Quantum Entanglement

In the above section, we have focused on realization of conventional functional devices in a nanometer space. Note that our proposed functional devices consist of two parts; coherent operation part with matter coherence and dissipative output part as illustrated in Fig. 17 or Fig. 4. The matter coherence in the coherent operation part is maintained for the period that exciton population moves to the dissipative output part. In other words, the output quantum dot acts as a selector to identify a certain quantum state by using resonant energy transfer and spatial symmetry in a quantum-dot system. Therefore, such a system is useful for detecting some information about quantum entanglement. In this section, we propose a special signal splitting device regarding to quantum-entangled states. To use quantum mechanical
Fig. 35. Temporal evolution of exciton population for the two-exciton state. The solid and dashed curves represent for the anti-symmetric and symmetric states, respectively. The coupling strength between QD-A and B is set as $\hbar U = 89 \mu$eV, and that between QD-B and C is $\hbar U' = \hbar U_{BC} = 14 \mu$eV. The coupling between QD-A and C is assumed to be zero, $\hbar U_{CA} = 0$.

The process and classical dissipative process, simultaneously, is a novel concept inherently originating from nanophotonics.

Figure 36 shows schematic illustration of the device, in which two output terminal quantum dots, QD-C and D, exist. The QD-C is located symmetrically regarding to two identical quantum dots, and the upper energy level is set with a positive shift by $U$. The QD-D is configured maximally asymmetrically with a negative energy shift $\Delta \Omega = -U$. From the resonance conditions, QD-C can extract exciton population from the coherent operation part via the symmetric state, while QD-D resonantly selects the anti-symmetric state, where resonant energy transfer is allowed because symmetry breaking. (See Fig. 35) Therefore, this system can distinguish an initial quantum-entangled state, $|\phi\rangle = c_1|1\rangle_s + c_2|1\rangle_a$, and information of weight coefficients $c_1$ and $c_2$ is converted to optical frequency or wavelength of the far-field light, which is released after the spontaneous lifetime (several ns) of excitons.

Numerical results of the exciton population dynamics in the above system are given in Fig. 37, which is calculated by using the density matrix formalism. The top in Fig. 37(a) is the time evolution in the case of $|c_1|^2 = 2/3$ and $|c_2|^2 = 1/3$ as an initial condition, and the bottom is that of $|c_1|^2 = 1/3$ and $|c_2|^2 = 2/3$. Such initial quantum-entangled states can be established by using asymmetrically located optical near-field source against QD-A and B, or by connecting to some quantum computing devices. Both cases in Fig. 37 show that the exciton population on the energy levels in QD-C and QD-D well reflects the initial weight coefficients, where the horizontal gray lines in Fig. 37 indicate the expected values. Note that the deviation from the expected values becomes large, as the weight coefficient of symmetric state $c_1$ increases.
Quantum entangled initial state $c_1|\psi\rangle + c_2|\phi\rangle$

This is caused that the off-resonant energy transfer from symmetric state to the output state slightly occurs in the asymmetrically arranged quantum-dot system. In contrast, the off-resonant energy transfer from anti-symmetric state to the output state is completely forbidden, since QD-C is located symmetrically. Therefore, a part of the exciton population that should flow into symmetrically arranged system, moves into asymmetrically arranged QD-D. This results in the difference depending on these weight coefficients.

To use quantum mechanical and classical process is one of characteristic device operations inherent in nanophotonic devices. We suppose there are further useful applications to be realized. Progressive investigation will be expected from the system architecture viewpoint.

4.6 Summary

In this section, we have discussed nanophotonic functional devices in which a coherently-coupled state between optical near field and nanometric quantum dots are utilized. For a theoretical aspect, we have derived equations of motion for excitons by using the bases of appropriate coupled states, i.e., symmetric and anti-symmetric states, while we had solved them by using those of isolated quantum-dot states in Sec. 3. As a result, a principle and conditions for selective energy transfer from the coherently-coupled states to the output state have been analytically shown. The selective energy transfer originates from two important factors; one is whether the coherently-coupled state is symmetric or anti-symmetric, where the two states have different eigenenergies. In addition, it is a characteristic feature that optical near field can access the anti-symmetric state, which is impossible for the far-field light to excite because the anti-symmetric state has dipole inactive nature. The other is
that we can control exciton energy transfer by designing spatial arrangement of several quantum dots; in the case of symmetrically arranged system, anti-symmetric state is dipole inactive, while asymmetrically arranged system, the exciton population in the anti-symmetric state can be attracted intently by adjusting the system asymmetry. By using these factors and preparing a larger quantum dot for an output terminal, some functional operations can be realized. As an analytical demonstration, we have proposed AND- and XOR-logic gates, and investigated their dynamics and system tolerance due to spatial asymmetry. Consequently, the AND-logic gate can be achieved by shifting the output energy level negatively, while the XOR-gate can by shifting that positively, where the energy shifts are determined by the optical near-field coupling strength, i.e., ±U. When the number of quantum dots increases, we can realize higher degree functional devices, although the selectivity or resonance conditions becomes more critical. As an example, we
have proposed a controlled-type logic gate by using four quantum dots, in which the AND and XOR-logic operations are exchanged in the same device depending on the number of incident excitons.

The above logic operations have been achieved by designing the system completely symmetrically, and adjusting the output energy level to the resonance conditions. A system with spatial asymmetry is very interesting from the following viewpoints; we can create a characteristic state which decouples from the far-field light. Such a state expects to have longer population lifetime than usual exciton lifetime time (spontaneous emission time), and thus, we can store photon energy or exciton-polariton energy in the system. It is useful for a buffer memory which directly stores photon energy. We have shown the operation to create a pure anti-symmetric state by using a maximally asymmetrically arranged three-quantum-dot system. For another viewpoint, we can realize an information processing device in which mixed states between symmetric and anti-symmetric states are utilized. We have proposed a device which identifies quantum entangled states by using the spatial symmetry and resonance conditions. This kind of device is different from simple quantum information processing devices. Both of quantum mechanical and classical information processing can be applied in suitable situations, which is one of inherent operations in nanophotonic devices.

In Sec. 3, although we had discussed a nanophotonic switch (equivalent to an AND-logic gate), where only energy states or resonance conditions have been considered, nanophotonic functional operations discussed in this section are realized by intentionally using the spatial degree of freedom, which is one of key features in nanophotonics. With future progress of nanofabrication techniques, where we can control materials, size, and position arbitrarily in a nanometer space, nanophotonics and its device architecture have a large possibility for expanding optical and electronic device technologies. We expect that the readers notice a part of such possibility in nanophotonics from discussions in this section.

5 Conclusions

In this Chapter, we have theoretically investigated operation principles of typical nanophotonic devices and their dynamics as well as formulated characteristic interaction between nanometric objects and nanometric light. The nanophotonic devices discussed here are based on several nanophotonic inherent features, such as local excited states, unidirectional energy transfer, and exciton number dependence. In Sec. 2, we have formulated energy transfer between two quantum dots by using exciton-polariton picture, and estimated strength of optical near-field coupling and energy transfer time. Moreover, we theoretically proved that the optical near-field interaction enables to excite a dipole-inactive energy level for the far-field light. The energy transfer rate or optical near-field coupling strength can be expressed by the overlap...
integral of envelope functions for two quantum dots and spatial spreading of
the optical near field which is described by a Yukawa function in the lowest
perturbation. Therefore, we find that the dipole-inactive state can excite
easier as inter-quantum-dot distance becomes smaller.

Using the results in Sec. 2, characteristic nanophotonic devices, which
consist of several quantum dots, have been proposed in Secs. 3 and 4, and
we have formulated and analyzed exciton dynamics by using quantum me-
chanical density matrix formalism. In Sec. 3, a nanophotonic switch has been
numerically demonstrated. In the switch, we use resonant and unidirectional
energy transfer, and the unidirectional energy transfer path changes due to a
fermionic interaction of excitons in the lowest energy level, i.e., a state-filling.
Our numerical estimation shows that the state-filling time is sub-100 ps for a
CuCl quantum-cube system. Furthermore, recovery time to the initial condi-
tions can be improved in the same order of the inverse of the optical near-field
coupling by using stimulated absorption and emission process for coupling to
the far-field reservoir. In other words, fast recovery time is achieved by adjust-
ing illumination power and pulse width of control light, which is independent
of spontaneous emission lifetime.

In addition to the operation principles in Sec. 3, we have proposed other
kind of nanophotonic devices which adopt symmetry of exciton excited states
as well as spatial arrangement of quantum dots as a novel degree of freedom.
In such devices, the signal can be selectively extracted by adjusting the en-
ergy level in the output quantum dot, as the output energy level is resonant
to the coherently coupled states in the input system with several quantum
dots. These are useful for a multi-input computation. We have shown re-
alization of AND and XOR-logic gates by using a symmetrically arranged
three-quantum-dot system. In these device operations, it is a key feature
that the resonance conditions are different depending on the exciton number.
Based on the similar principle, we show possibility for a higher-degree func-
tional device, such as a controlled-type AND and XOR-logic gates, by using
more number of quantum dots. On the other hand, resonant energy transfer
via an anti-symmetric state becomes possible in an asymmetrically arranged
system. Although the anti-symmetric state cannot be excited by the far-field
light, simultaneously, this is also the state which cannot radiate the far-field
photons. We have shown that the anti-symmetric state is intentionally cre-
bated by using the optical near-field coupling and its spatial asymmetry. This
indicates that the photon energy can be stored in the system as the form of
an exciton-polariton. In principle, a photon buffer memory can be realized
by these mechanism. To extract the symmetric and anti-symmetric states
selectively is equivalent to identify some quantum-entangled states, since the
bases are superposition of several quantum-dot excitations. Therefore, the
asymmetry of the system is useful for an interface device which manipulates
quantum-entangled states. As an example, we have numerically demonstrated
that quantum-entangled states with the symmetric and anti-symmetric states
can be separately detected in the symmetrically and asymmetrically arranged quantum dots, which means that the quantum-entangled states are distinguished by the output photons with different energies. Although, in the field of quantum information processing, only parallel information processing is attracted, a device, which intentionally lowers the coherence or partially uses it, has large possibility to extend a viewpoint in conventional device technologies.

In the above, our discussions have been focused in the theoretical aspects. Experimental studies for nanophotonic devices have been intensively advanced. A nanophotonic switch discussed in Sec. 3 has already verified experimentally [30]. Our theoretical study has been used for fitting into the experimental data and for evaluation of physical constant, such as the strength of optical near-field coupling. Furthermore, some interesting devices have been also proposed and investigated. For example, so-called nanofountain [42], which a nanophotonic summation device has been successfully demonstrated, and in such device, the unidirectional energy transfer is utilized. On the other hand, a devices, in which coherently coupled states are used, has not reported as far as we know. However, there are some fundamental studies related to the coherently coupled state; for example, presence of anomalous long lifetime, which originates from an anti-symmetric state, has been discussed theoretically and experimentally [43,44]. For realization of such a device using spatial degree of freedom in a nanometric space, further progress of nanofabrication technology is required. However, nanofabrication technology has been developed recently [45,46].

Finally, we like to spare the remaining for the future outlook. All of our investigation about nanophotonic devices are based on the quantum mechanical density matrix formalism, which is the manner to describe averaged dynamics. We anticipate that an ultimate operation of nanophotonic devices is input and output of carrier energies with single photon level. In order to realize such device, theory of optical near-field interaction must be rewritten in the form including a photon statistics rigorously, which is rather interesting problem to be solved in the near future. Furthermore, a novel possibility for nanophotonic device technology may be hidden in such consideration. Research in nanophotonics and nanophotonic device technology is still at a starting line, and we expect them further innovatively developed, which we cannot speculate.

Acknowledgements

The authors are grateful to T. Yatsui of Japan Science and Technology Agency, T.-W. Kim of Kanagawa Academy of Science and Technology, H. Hori, and I. Banno of Yamanashi University for fruitful discussions. This work was carried out at the project of ERATO, Japan Science and Tech-
technology Agency, since 1999. The authors would like to thank for the persons concerned.

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